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AN EXTENSION OF THE WEAR MODEL FOR THE CUTTING TOOL DURING MACHINING A WINKLER BRITTLE MATERIAL

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Abstract

The wear of a cutting tool is the most important parameter which defines the tool durability; this parameter has a big influence on the quality of a working piece, tool vibration and the dimensional position.
The mechanical properties of the working material and the geometry of the cutting tool define the pressure distribution on the contact surface and the cutting forces.
The Winkler property of a working piece is defined by that the local deformations are proportional with the pressure in one point. These deformations are independent on the neighbor pressure.
Starting from the several researches about the variation of the cutting tool shape and about the shape of the Winkler brittle material, during the machining process, it was realized an extension of these researches, analyzing the pressure distribution, the cutting forces and the profile of the worn cutting tool.

Keywords: *wear, cutting tool, pressure distribution, worn profile, brittle material.*

1. INTRODUCTION

The endurance of the cutting tool is influenced by its wearing. This parameter can be determined taking into account the pressure action on the cutting sides of the cutting tool during manufacturing process but also its geometry.
This paper tries to make an extension of the wearing model of a cutting tool in order to manufacture fragile Winkler materials.
The Winkler brittle material is characterized by local deformations which are proportional with the pressure existing in the respective point and are independent of nearby pressures.

2. CONTACT PRESSURE MODEL

It is considered a cutting tool under the form of a turning cutting tool. The analysis starts from the simplified hypothesis that considers the cutting tool as being bi-dimensional, as the third dimension does not influence in a significant way the process that will be analyzed further on.

In figure 1 is presenting the geometrical model for the tool, characterized by angles β and γ.

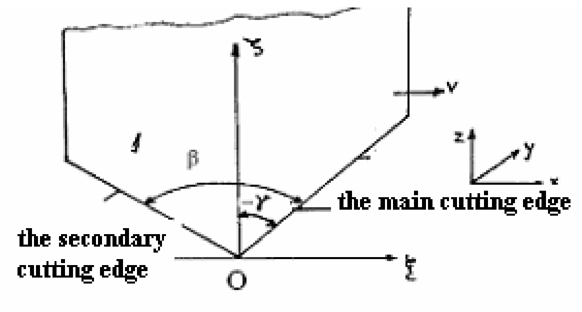

Figure1: *The tool model*

Attaching to the tool the axis system (ξ,ζ) having the origin O and to the work piece the axis system (x, y, z), it is obtained:

$$x = \xi + vt \qquad z = \zeta - c(t) \tag{1}$$

where: v - cutting speed;
c(t) - cutting depth;
t – time.

In the axis system (ξ,ζ), the geometry of the tool nose is characterized by the function f(ξ,t). In the cutting process the tool deform elastically the workpiece under the critical cracking pressure p^*.

The elastically deformation (w(x,t)) is proportional with the material pressure ($\vec{p}_2(x,t)$):

$$w(x,t) = k\vec{p}_2(x,t) \tag{2}$$

where: k – the material elasticity characteristic:

$$k = \frac{(1+\nu)(1-2\nu)h}{(1-\nu)E} \tag{3}$$

where: ν- Poisson's coefficient;
E – elastic modulus;
h - material layer thickness.

The contact condition of the tool with the working aria, f(ξ,t) can be written under the differential form:

$$k_1\vec{p}_2(x,t) + k\frac{d\vec{p}_2(x,t)}{dt} = \frac{dc(t)}{dt} - \frac{df(x-vt,t)}{dt} \tag{4}$$

In the coordinate system (ξ,ζ), the equation (4) becomes:

$$k_V p(\xi,t) + k\left(\frac{\partial p(\xi,t)}{\partial t} - \frac{\partial p(\xi,t)}{\partial \xi}v\right) = \frac{dc(t)}{dt} + \frac{\partial f(\xi,t)}{\partial \xi}v - \frac{\partial f(\xi,t)}{\partial t} \tag{5}$$

where: $p(\xi,t) = \vec{p}_2(\xi + vt, t)$
k_v - speed coefficient of the tool.

The wearing speed of the tool is considered to be depended on the result obtained by multiplying the sliding speed (v) with the normal pressure on the contact surface (p_n):

$$\frac{\partial f(\xi,t)}{\partial t} = k_W p_n(\xi,t)\cdot\frac{v}{\cos(\beta_n)} \tag{6}$$

where: k_w - wearing coefficient of the tool material;

Based on the differential geometry elements, the angle β_n can be defined using the derivative function f(ξ,t), $tg\beta_n = \frac{\partial f(\xi,t)}{\partial \xi}$.

Therefore the active profile of the cutting tool varies in the wearing process according to the relation:

$$\frac{\partial f(\xi,t)}{\partial t} = k_W p(\xi,t)v\left[1 + \left(\frac{\partial f(\xi,t)}{\partial \xi}\right)^2\right] \tag{7}$$

For manufacturing a Winkler material type, the contact pressure on the tool can be determined following the differential equation (5). Then, the form of the worn-out profile of the tool is evaluated (equation 7), considering the cutting process stationary, with the workpiece-cracking and wearing of the cutting tool.

3. DETERMINATION OF THE PRESSURE DISTRIBUTION ON THE EDGES OF THE CUTTING TOOL

3.1. Stationary process with workpiece-cracking and wearing of the cutting tool (sharp nose)

The case of the angular cutting tool is considered as having the wearing speed under the form of:

$$\frac{\partial f(\xi,t)}{\partial t} = \begin{cases} k_W p(\xi,t)v\left(1+m_{u1}^2\right) & \text{for } \xi \le 0 \\ k_W p(\xi,t)v\left(1+m_{u2}^2\right) & \text{for } \xi > 0 \end{cases} \tag{8}$$

where: $m_{u1}=\tan(\pi/2+\beta-\gamma)$ - the angular coefficient of the secondary edge;

$m_{u2}=ctg(\gamma)$ - the angular coefficient of the principal edge.

Taking this into account the solution of the differential equation (5), having the unknown data $p(\xi,t)$, become:

$$p_a=\begin{cases} A_{12}+B_{12}\exp\left(\frac{1}{B_{12}}\xi_a+ct_{12}\right) \\ \text{for } \xi_a \le 0 \\ A_{22}+B_{22}\exp\left(\frac{1}{B_{22}}\xi_a+ct_{22}\right) \\ \text{for } \xi_a > 0 \end{cases} \quad (9)$$

where:

$$A_{12}=\frac{c_0+\frac{m_{u1}}{kp^*}}{\frac{1}{kv}\left[k_v+k_w v\left(1+m_{u1}^2\right)\right]}$$

$$B_{12}=\frac{1}{\frac{a}{kv}\left[k_v+k_w v\left(1+m_{u1}^2\right)\right]}$$

$$A_{22}=\frac{c_0+\frac{m_{u2}}{kp^*}}{\frac{1}{kv}\left[k_v+k_w v\left(1+m_{u2}^2\right)\right]}$$

$$B_{22}=\frac{1}{\frac{a}{kv}\left[k_v+k_w v\left(1+m_{u2}^2\right)\right]} \quad (10)$$

where: c_0 – constant stress;

a – constant value on the ξ axe.

Integration constants are determined to the limit: $p_a(1)=p_a(b_{a2})$, where $b_{a2}=b_2/a$, (b_2 is ξ-coordinate of the contact point on the secondary edge).

The pressure distribution is obtained using the expression:

$$p_a(\xi_a)=\begin{cases} A_{12}\left[1-\exp\left(\frac{\xi_a-b_{a2}}{B_{12}}\right)\right]=p_{a1} \\ \text{for } \xi_a \le 0 \\ A_{22}\left[1-\exp\left(\frac{\xi_a-1}{B_{22}}\right)\right]=p_{a2} \\ \text{for } \xi_a > 0 \end{cases} \quad (11)$$

The a dimensional abscissa on the secondary cutting side of the cutting tool is deduced out of the condition of pressure continuity in point $\xi_a=0$:

$$b_{a2}=-B_{12}\ln\left\{1-\frac{A_{22}}{A_{12}}\left[1-\exp\left(-\frac{1}{B_{22}}\right)\right]\right\} \quad (12)$$

In figure 2 is presented the distribution of non dimensional pressures $p_a(\xi_{a1}, \xi_{a2})$ for different values of the wearing coefficient k_w and of the speed parameter $v_a= v/p^*k_v$; $p_a=p/p^*$.
(v_a = 0.1, 0.5, 10; k_w = 0.0000001, 0.000001, 0; β = 70°, γ = 15°; $p^*= 10^4$ MPa)

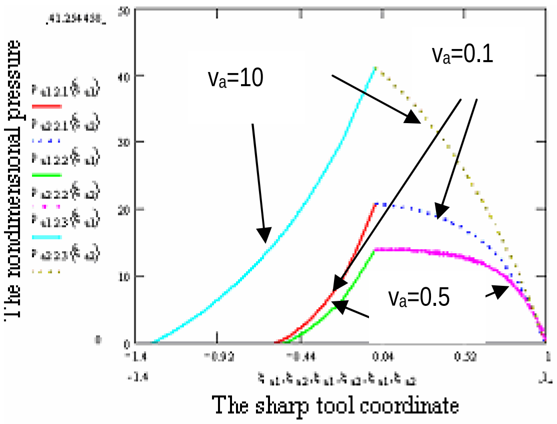

Figure 2: *Pressure distribution on active faces of the tool*

3.2. Stationary process with workpiece-cracking and wearing of the cutting tool (rounded nose)

It is considered a tool characterized by angles β and γ and the connection radius r between the two edges (figure 3).

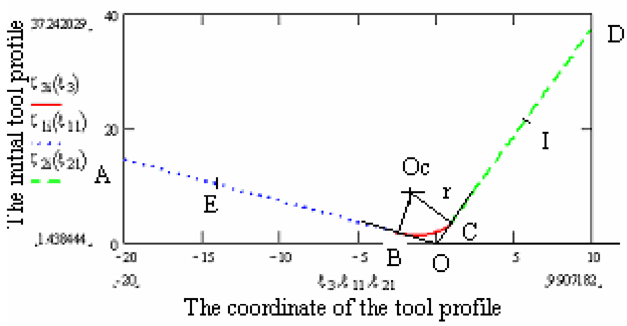

Figure 3: *The shape of the rounded tool (β =70', γ =15', r =0.4)*

In this case, the differential pressure equation will have different expression for the three areas of the cutting edges: EB-secondary cutting edge, BC-rounded end angle of the tool, CT-main cutting edge.

The solution of the differential equation (5), in this case, for the intervals $\xi_a \in [\xi_{aE}, \xi_{aB}]$ and $\xi_a \in [\xi_{aC}, \xi_{aI}]$ is:

$$p_a(\xi_a) = A_{12}\left[1 - \exp\left(\frac{\xi_a - \xi_{aE}}{B_{12}}\right)\right]$$

$$p_a(\xi_a) = A_{22}\left[1 - \exp\left(\frac{\xi_a - 1}{B_{22}}\right)\right] \tag{13}$$

For the interval $\xi_a \in [\xi_{aB}, \xi_{aC}]$, the solution can be written under the form:

$$p_a(\xi_a) = \exp\left[\int_{\xi_{aE}}^{\xi_a} g_1(\xi_a)d\xi_a\right] \cdot \left\{\int_{\xi_{aE}}^{\xi_a} g_2(\xi_a)\exp\left[-\int_{\xi E}^{\xi_a} g_1(\xi_a)d\xi_a\right]d\xi_a + ct_3\right\} \tag{14}$$

where: $g_1(\xi_a) = \frac{a}{kv}\left[k_v + k_w v\left(1 + g^2(\xi_a)\right)\right]$

$$g_2(\xi_a) = -\frac{c_0}{kv}\frac{a}{p^*} - \frac{av}{kvp^*}g(\xi_a)$$

The integration constant ct_3 and the integration limit ξ_{aE} are determined from the continuity condition of pressure in points C and B:

Therefore we obtain:

$$ct_3 = A_{12}\left[1 - \exp\left(\frac{\xi_{aB} - \xi_{aE}}{B_{12}}\right)\right]$$

$$\xi_{aE} = \xi_{aB} - B_{12}\ln\left\{1 - \frac{A_{22}\left[1 - \exp\left(\frac{\xi_{aC} - 1}{B_{22}}\right)\right]}{A_{12}\exp\left[\int_{\xi_{aB}}^{\xi_{aC}} g_1(\xi_a)d\xi_a\right]} + \frac{1}{A_{12}}\int_{\xi_{aB}}^{\xi_{aC}} g_2(\xi_a)\exp\left[-\int_{\xi_{aB}}^{\xi_{aC}} g_1(\xi_a)d\xi_a\right]d\xi_a\right\} \tag{15}$$

The equation (14.), determined using constants ct_3 and ξ_{aE} (obtained from the relation 15 using MathCAD program) is used for representing the pressure distribution on BC area of the tool nose (figure 4).

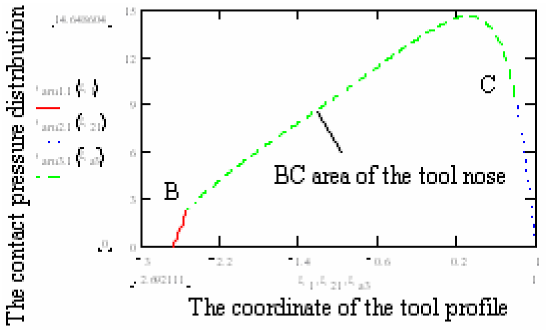

Figure 4: *Pressure distribution to the nose of the rounded tool*

4. DETERMINATION OF WORN OUT PROFILE OF THE CUTTING TOOL

4.1. The case of completly sharped tool

From the equation (9), taking in to account the wearing speed under the form (8), was deduced the differential equation, having the unknown data $p(\xi,t)$

$$\frac{dp_a}{d\xi_a}(\xi_a, t_a) = \frac{a}{kv}\left[k_v + k_w v\left(1 + m_{u1,2}^2\right)\right]p_a - \frac{c_0}{kv}\frac{a}{p^*} - \frac{m_{u1,2}}{k}\frac{a}{p^*} \tag{16}$$

with $\xi_a = \xi/a$; $t_a = t/t^*$ and $p_a = p/p^*$

Integrating correspondently with the time, it is obtained the equation of the relative profile ($f = f/a$):

$$f_a(\xi_a, t_a) = \begin{cases} f_{oa}(\xi_a) + k_w p^* t_a\left(1 + m_{u1}^2\right)p_{a1}(\xi_a) & \text{for } b_{a2} < \xi \le \xi_{as1}^* \\ f_{oa}(\xi_a) + k_w t_a\left(1 + m_{u1}^2\right) & \text{for } \xi_{as1}^* \le \xi \le 0 \\ f_{oa}(\xi_a) + k_w t_a\left(1 + m_{u2}^2\right) & \text{for } 0 \le \xi \le \xi_{as2}^* \\ f_{oa}(\xi_a) + k_w p^* t_a\left(1 + m_{u2}^2\right)p_{a2}(\xi_a) & \text{for } \xi_{as2}^* < \xi \le 1 \end{cases} \tag{17}$$

where: $t_a = t/t^*$ is the non dimensional contact time of the tool with the material characterized by critical pressure p^*, elasticity parameter k and settling parameter k_v.

In figure 5 is presented the form of the worn tool:

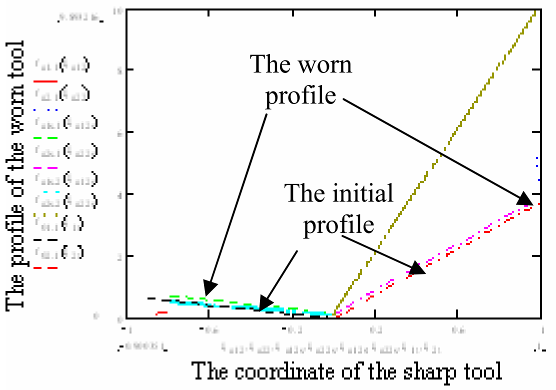

Figure 5: *The profile of the worn-out cutting tool*

4.2. The case of the rounded tool nose

The form of the worn out profile of the tool $f_a(\xi_a,t_a)$ is determined integrating the equation:

$$\frac{\partial f_a(\xi_a,t_a)}{\partial t_a}=$$

$$k_w p^* p_a(\xi_a,t_a)v\left[1+\left(\frac{\partial f_a(\xi_a,t_a)}{\xi_a}\right)^2\right]=$$

$$=\begin{cases} k_w p^* p_a(\xi_a,t_a)v\left[1+m_{u1}^2\right] \\ \text{for } \xi_{aE}\le\xi_a\le\xi_{aB} \\ k_w p^* p_a(\xi_a,t_a)v\left[1+m_{u2}^2\right] \\ \text{for } \xi_{aC}\le\xi_a\le 1 \\ k_w p^* p_a(\xi_a,t_a)v\cdot \\ \cdot\left\{1+\left[\frac{\xi_a-\xi_{aO_c}}{\sqrt{r_a^2-(\xi_a-\xi_{aO_c})^2}}\right]^2\right\} \\ \text{for } \xi_{aB}<\xi_a\le\xi_{aC} \end{cases} \quad (18)$$

Considering that at the moment t =0, the profile is the initial one:

$$f_a(\xi_a,t_a)=f_{oa}(\xi_a)+$$
$$+k_w p * p_a(\xi_a)t_a\left[1+\left(\frac{\partial f_a(\xi_a)}{\partial\xi_a}\right)^2\right] \quad (19)$$

In figure 6 is presented the profile of the worn-out tool for BC area of the tool nose:

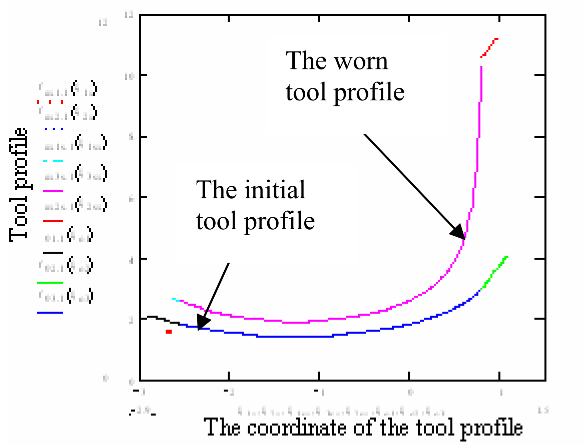

Figure 6: *The shape of the worn-out tool*

5. CONCLUSIONS

Analyzing the pressure distribution and the profile of the worn cutting tool in both cases of the tool nose(sharp and rounded) it is obtained some important conclusions:

- The maximum pressure point is situated around the C point (figure 4), where the wear is maximum.
- The maximum pressure contact point corresponds with the maximum wear point, even the contact pressure is bigger than the critical Winkler material pressure;
- The wear of the cutting tool can be evaluated by the modifying of the cutting angle, which involves the decrease of the cutting productivity;
- The optimization of the tool life can be realized if are correctly chosen the cutting angle, the value of the rake angle and the tool shape.

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