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A NEW APPROACH FOR DETERMINING THE FRICTION LAW IN METAL FORMING

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Abstract

The most popular friction law in metal forming applications is Tresca's law which postulates that the friction stress is equal to a portion of the local shear yield stress. The friction factor involved in this law should be found from experiment. A typical test for this purpose is the ring compression test. The test must be supplemented with a theoretical analysis. The latter is often based on upper bound solutions. There are several difficulties with the interpretation of test results. The distribution of the friction factor over the surface is not uniform, though it is assumed to be uniform in the theoretical analyses. It is very difficult (or even impossible) to minimize this non-uniformity to a reasonable level in experiment. It is rather necessary to develop theoretical techniques to account for it in inverse problems where the frictional boundary condition should be obtained as a result of the theoretical analysis and experimental data. Another difficulty is that sticking can occur on a part of the friction surface. This actually happens in ring compression tests, and usually is not accounted for in theoretical analyses. This difficulty is specific to the ring compression test (and other similar tests) and can be overcome by selecting an appropriate test. Such a test is proposed in the present paper. Dies have the form of a cone. The specimen is a pre-shaped hollow cylinder such that the die exactly fits the cavity made at each cylinder face. The new test reduces to the ring compression test in a limit. The cone angle should be chosen such that no sticking occurs. In the paper, an upper bound solution is proposed to supplement experimental data. Using this solution, conclusions are drawn on the applicability of the test for determining the friction factor.

Key words: *Friction Law, Metal Forming, Ring Upsetting by Conical Dies*

INTRODUCTION

In the context of plastic forming, contact friction represents the resistance to the relative movement of specimen material along the contact surface of the tool. This, in principle, has negative impact on the forming process – causing increase of forming load and deformation work, increase unhomogeneity of stress and strain distribution in specimens, decrease of material formability, increase of tool wear and shortening of its operating life. Rolling is the only process where contact friction has positive effects, as it facilitates introduction of material into the deformation zone.

In the plastic forming technology, the nature of friction is substantially different to that of the conventional mechanical assemblies, primarily due to much higher pressure – which, in plastic forming, can amount to 2.500 MPa. This is why different laws of friction apply in plastic forming technology [1].

Although, in some cases without the real justification, Coulomb friction law is applied in bulk forming technologies, as well as in metal sheet forming processes. According to this law, there exists a proportionality between the normal contact stress (σ_n) and the friction coefficient (μ):

$$\tau_k = \mu \cdot \sigma_n \tag{1}$$

This law is applied in processes of cold forming where the normal stress (σ_n) is lower then the flow stress (K).

Tresca's friction law [1] is applied in cases when the normal contact stress is higher then the flow stress, i.e. when specimen material has a tendency of sticking to the tool surface. In that case, the tangential contact stress is proportional to the friction factor (m) and the local shear stress (τ_{max}):

$$\tau_k = m \cdot \tau_{\max} \tag{2}$$

Friction factor is related to friction coefficient by $m = \sqrt{3} \cdot \mu$, where the friction factor ranges between $0 \le m \le 1$ and the friction coefficient ranges between $0 \le \mu \le 1/\sqrt{3}$.

For the processes of rolling, wire drawing and deep drawing as well as for the processes of ironing, dimpling and expanding, the following friction law is applied:

$$\tau_k = \mu \cdot K \tag{3}$$

assuming that the normal contact stress is lower than the flow stress.

The lessening of the negative friction impact in plastic forming technology is performed by lubrication, using special means and methods, without which the practical application of some technologies (e.g. cold extrusion) would be impossible.

Various theoretical and experimental methods are used for determination of the friction coefficient (μ), i.e. friction factor (m). J. Scheya's book [1] reviews the methods for determination of friction forces and friction coefficient for various processes of bulk and sheet metal forming. Methodology for contact friction determination is based on measurement of process parameters and calculation of tangential contact stress, i.e. friction coefficient, based on the theoretical background of the process in hand.

For the processes of rolling, wire drawing and extrusion, determination of friction coefficient is based on the measurement of the total load and friction force, that is, on the measurement of normal and tangential contact stress using special tools, i.e. tools with built-in sensors. These processes often require application of a special pin load cell to measure contact stress and friction coefficient.

According to [1], friction coefficient and friction factor for upsetting and cold and warm forging are determined using the following methods: cylinder upsetting by flat plates, cylinder upsetting by conical-convex tools, plate upsetting by flat tools in plane strain condition, pin load cell method and ring upsetting method.

The method of pin load cell relies on measuring the normal and tangential contact stress in a small area, which is used to calculate the friction coefficient. This method requires highly sensitive measuring devices to be built into the tool. The contact stress is, via pin transferred to the electronic dynamometer, which enables measurement of contact stress during forming. Determination of contact friction requires at least two, i.e. three pin load cell measuring instruments, depending on the stress-strain state in the specimen. Paper [3] documents the process of contact friction coefficient determination in prismatic specimen upsetting with cylindrical tools. In the upper tool, the pin load cell is placed in the radial direction, while in the lower tool, it is offset 30^0 relative to the radial direction, which enables determination of normal and tangential contact stress and the friction coefficient. The rotation of cylindrical tools prior to the beginning of the forming process, allows the position of the pin load cells to be changed, thus allowing contact stresses to be measured along the whole contact area. Illustrated in paper [4] is the method for determination of friction coefficient for warm forming. A measuring system with three pin load cells is used, where the pins are insulated from the electronic dynamometer by a zirconium rod, in order to prevent the heat transfer to the dynamometer. In this way, the measuring accuracy of the friction coefficient is unaffected by the thermal influence. Special problem with the method of pin load cell represent the calibration of the measuring system and the impact of clearance between the pin and its housing on the measuring accuracy. The measuring result is also affected by the starting position of the pin load cell relative to the tool contact surface.

One of the most frequently used methods for determining friction factor and coefficient of friction in cold and warm bulk forming, is the method of ring upsetting. Kunogi first introduced this method for cold forming processes, while it was later improved by Male and Conckroft for warm bulk forming [5]. The method is based on monitoring changes in ring geometry as compared to the calibration diagram, which represents the ratio between strains of the inner ring diameter and ring height for various friction coefficients. This method is simple to apply, demanding simple specimen and tool. Presented in [6] is the analysis of influence of initial ring geometry on the calibration diagram, which was conducted by the upper bound solution [2]. This analysis has established that the initial ring dimensions influence the flow of material and the form of the calibration diagram. However, the friction coefficient does not depend on the initial ring dimensions, assuming the adequate calibration diagram is used. For the purpose of friction coefficient determination, it is a common practice to use rings whose initial dimension ratio *outer diameter:inner diameter:height* is 6:3:2, respectively. For different classes of materials the test has been extended in [7, 8].

Presented in this paper are the theoretical foundations of shear coefficient determination, using method of ring upsetting by conical tools. The main purpose of the theoretical solution is determine such process parameters that no sticking zone occurs.

STATEMENT OF THE PROBLEM

The geometry of the process, the cylindrical coordinate system r, φ, z and the special coordinate system ρ, φ, θ are illustrated in Fig. 1. A quarter of the specimen gross-section is shown. A hollow cylinder compressed between two conical dies moving along the z axis with a velocity U. The geometry of the specimen is completely determined by the following four parameters: h, H, R_0 and R (Fig. 1). Then, it follows from geometrical considerations that

$$\tan\alpha = \frac{H-h}{R-R_0} \quad \textit{and} \quad s = h\cot\alpha - R_0 \tag{4}$$

where α and s are defined in Fig. 1. Owing to the symmetry, it is possible to consider a half of the specimen at $z \geq 0$. The transformation equations between the two coordinate systems are

$$r = \rho\cos\theta - s, \quad z = \rho\sin\theta, \quad \varphi = \varphi \tag{5}$$

Using (5) the scale factors for the coordinate curves of the system ρ, φ, θ can be found in the form

$$h_\rho = 1, \quad h_\theta = \rho, \quad h_\varphi = \rho\cos\theta - s \tag{6}$$

Then, the non-zero components of the strain rate tensor in the coordinate system ρ, φ, θ are expressed as

$$\xi_{\rho\rho} = \frac{\partial u_\rho}{\partial\rho}, \qquad \xi_{\varphi\varphi} = \frac{u_\rho\cos\theta - u_\theta\sin\theta}{\rho\cos\theta - s}, \tag{7}$$

$$\xi_{\theta\theta} = \frac{1}{\rho}\left(\frac{\partial u_\theta}{\partial\theta} + u_\rho\right), \qquad \xi_{\rho\theta} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial\rho} + \frac{1}{\rho}\frac{\partial u_\rho}{\partial\theta} - \frac{u_\theta}{\rho}\right)$$

where u_ρ and u_θ are the velocity components in the ρ- and θ- directions. The circumferential velocity and the strain rate components $\xi_{\rho\varphi}$ and $\xi_{\theta\varphi}$ vanish due to the axial symmetry. Then, the equivalent strain rate for the problem under consideration is defined as

$$\xi_{eq} = \sqrt{\frac{2}{3}\left(\xi_{\rho\rho}^2 + \xi_{\theta\theta}^2 + \xi_{\varphi\varphi}^2 + 2\xi_{\rho\theta}^2\right)} \tag{8}$$

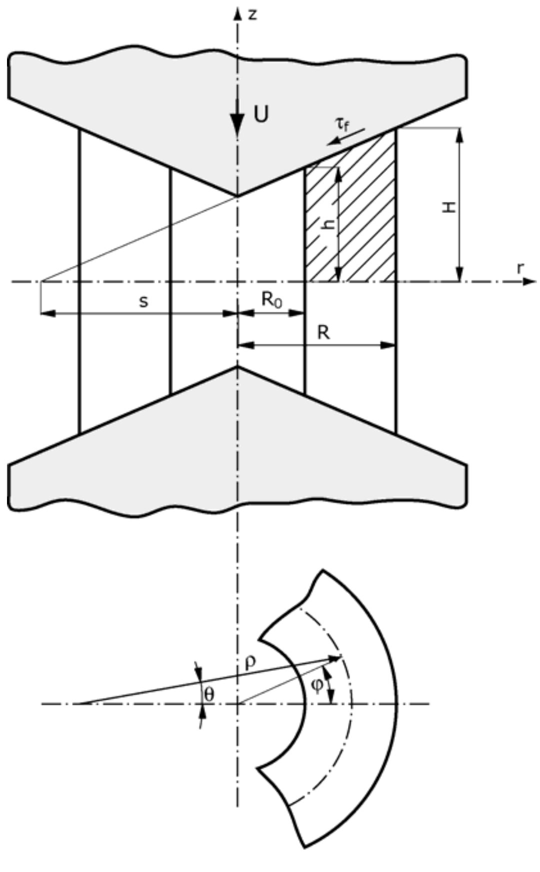

Figure 1: *Geometry of specimen and coordinate systems*

The velocity boundary conditions are

$$u_\theta = -U\cos\alpha \tag{9}$$

at $\theta = \alpha$ and

$$u_\theta = 0 \tag{10}$$

at $\theta = 0$. The stress boundary conditions consist of the friction law in the form

$$\tau_f = mk \tag{11}$$

at $\theta = \alpha$, the symmetry condition

$$\sigma_{\rho\theta} = 0 \tag{12}$$

at $\theta = 0$ and the condition that the surfaces $r = R_0$ and $r = R$ are stress free. Here τ_f is the friction stress, k is the shear yield stress, m is the friction factor, $0 \le m \le 1$, $\sigma_{\rho\theta}$ is the shear stress in the coordinate system ρ, φ, θ.

Taking into account the stress boundary conditions the upper bound theorem for rigid perfectly/plastic material reads

$$\begin{gathered} PU \le 2\sqrt{3}\pi k \int_0^\alpha \int_{\rho_1(\theta)}^{\rho_2(\theta)} \xi_{eq} h_\rho h_\theta h_\varphi d\rho d\theta \\ + 2\pi mk \int_{\rho_{\min}}^{\rho_{\max}} \left| u_\rho \big|_{\theta=\alpha} \right| d\rho \end{gathered} \tag{13}$$

where P is the compression force and u_ρ in the second integrand should be calculated at $\theta = \alpha$. Also,

$$\begin{gathered} \rho_1(\theta) = \frac{R_0 + s}{\cos\theta}, \quad \rho_2(\theta) = \frac{R + s}{\cos\theta}, \\ \rho_{\min} = \frac{R_0 + s}{\cos\alpha}, \quad \rho_{\max} = \frac{R + s}{\cos\alpha} \end{gathered} \tag{14}$$

The integrands in (13) can be calculated with the use of any kinematically admissible velocity field satisfying the velocity boundary conditions and the incompressibility equation. The latter has the form

$$\xi_{\rho\rho} + \xi_{\theta\theta} + \xi_{\varphi\varphi} = 0 \tag{15}$$

Kinematically admissible velocity field

Assume the following distribution of the velocity component u_θ

$$\frac{u_\theta}{U} = -\cot\alpha \sin\theta \tag{16}$$

In this form, the velocity u_θ satisfies the velocity boundary conditions (9) and (10). Substituting (7) into (15), with the use of (16), results in the following differential equation for the velocity u_ρ

$$\begin{gathered} \frac{\partial u_\rho}{\partial \rho} + \frac{u_\rho \cos\theta + U\cot\alpha \sin^2\theta}{\rho\cos\theta - s} \\ + \frac{u_\rho - U\cot\alpha\cos\theta}{\rho} = 0 \end{gathered} \tag{17}$$

The general solution to this equation is

$$\frac{u_\rho}{U} = \left(\frac{\rho^2}{2}\cos 2\theta - s\rho\cos\theta + \omega \right) \frac{\cot\alpha}{(\rho\cos\theta - s)\rho} \tag{18}$$

where ω is an arbitrary function of θ. Equations (16) and (18) determine a kinematically admissible velocity field at any function $\omega(\theta)$. The velocity u_ρ may vanish at a point $\rho = \rho_n$ of the friction surface, $\theta = \alpha$, which indicates that sticking regime can occur. Since $\rho\cos\theta - s > 0$ due to (5), equation (18) gives

$$\rho_n = \frac{s\cos\alpha \pm \sqrt{s^2\cos^2\alpha - 2\omega(\alpha)\cos 2\alpha}}{\cos 2\alpha} \tag{19}$$

In what follows, it is assumed, with no loss of generality, that $R = 1$ and $U = 1$.

Because of the symmetry of the problem with respect to the plane $\theta = 0$, $\omega(\theta)$ should be an even function of its argument. Such a choice of this function, when combined with the associated flow rule, satisfies the stress boundary condition (12). Even though it is not a requirement of the upper bound theorem, it is better to satisfy this condition to increase the accuracy of predictions. A simple function of this class is

$$\omega = a_0 + a_1\cos\theta \tag{20}$$

Substituting (20) into (18) it is possible to arrive at the kinematically admissible velocity field involving two undetermined parameters, a_0 and a_1. This field can be further substituted into equation (13), with the use of (7) and (8), giving the right hand side of (13) as a function of a_0 and a_1. Then, this function should be minimized with respect to a_0 and a_1 to obtain the best upper bound based on the velocity field chosen. This minimization has been performed numerically.

NUMERICAL RESULTS

To illustrate the theory developed, the numerical solution has been obtained assuming that $h = 0.7$, $H = 1$ and $R_0 = 0.5$. The variation of the ratio of the compression force to its value in the case of frictionless interface ($m = 0$) denoted by P_0 with the friction factor m is shown in Fig.2.

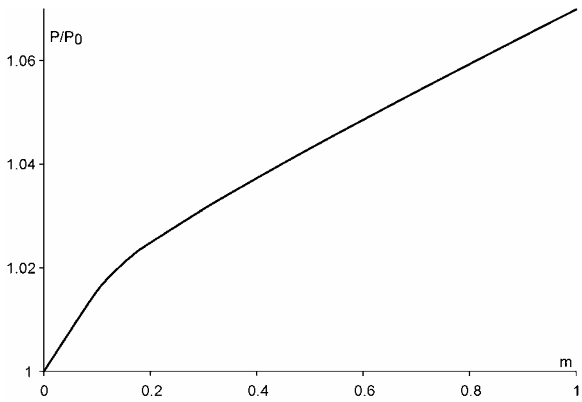

Figure 2: *Variation of compression force with the friction factor*

It is seen from this figure that the compression force slightly increases as the friction factor increases. The variation of the value of parameters a_0 and a_1 involved in equation (20) is depicted in Fig.3.

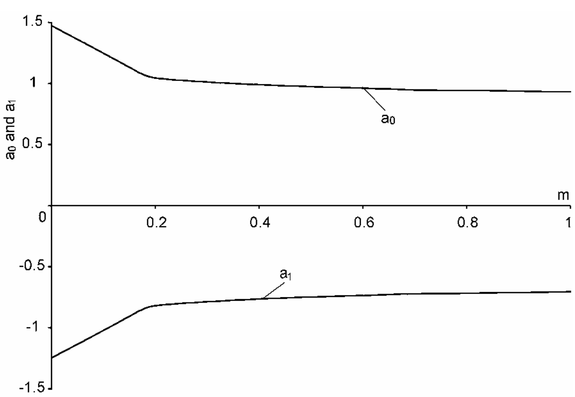

Figure 3: *Variation of parameters a_0 and a_1 involved in equation (20) with the friction factor*

Having these values, it is possible to calculate ρ_n with the use of (19) and (20), and the velocity u_ρ with the use of (18) and (20). In particular, it has been found that $\rho_n < \rho_{\min}$ at $m < 0.18$. It is seen from figures 3 and 4 that the curves have a high gradient in the vicinity of the point $m = 0.18$. The tendency of the shape of the free surfaces is determined by the velocity u_r at the free surfaces. The velocity u_r is expressed through the velocities u_ρ and u_θ as

$$u_r = u_\rho \cos\theta - u_\theta \sin\theta \qquad (21)$$

Using equations (16), (18) and (21) the velocity u_r as a function of z has been calculated at each free surface for several values of the friction factor. These distributions are shown in Figs. 4 and 5.

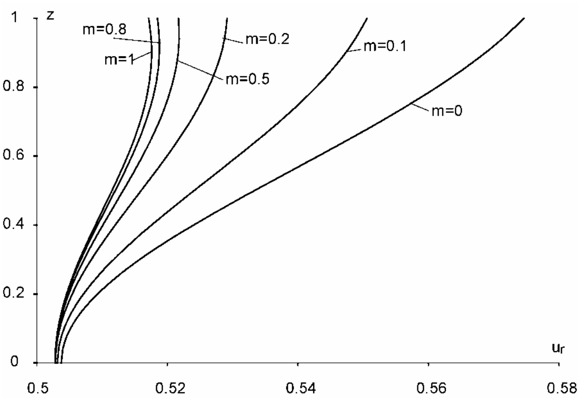

Figure 4: *Distribution of the radial velocity along z-axis at the free surface r = R*

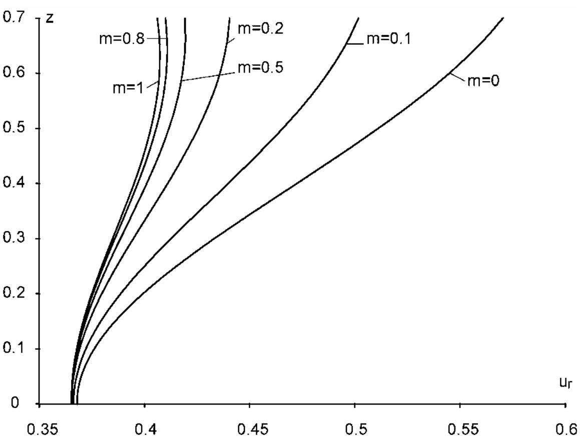

Figure 5: *Distribution of the radial velocity along z-axis at the free surface r = R_0*

For a better interpretation of experimental data, it is advantageous that $\rho_n < \rho_{\min}$ since it indicates that sliding can occur over the entire friction surface. Since the maximum possible value of m is 1, it is important to find such geometry of the specimen that $\rho_n < \rho_{\min}$ at $m = 1$. Changing the value of h, it has been found that the aforementioned condition is satisfied at $h < 0.5$.

CONCLUSIONS

An upper bound theoretical solution for compression of a hollow cylinder by conical dies has been proposed. It is expected that the solution can be used to interpret experimental data for determining the friction factor. An important feature of this upsetting process, as compared with compression between flat dies, is that it is possible to find such geometry of the specimen that no sticking zone occurs. It seems that it is important for determining the friction factor. The present solution introduces no rigid zone at the friction surface. Therefore, its predictions are very approximate and the solution, because of its simplicity and, can be used as a first approximation for more sophisticated solutions.

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REFERENCES:

[1] Schey J.: Metal Deformation Processes – Friction and Lubrication, Marcel Dakker Inc., New York, 1972.

[2] Avitzur B.: Metal Forming - Processes and Analysis, New York, 1968.

[3] Plančak M., Vilotić D: Application of Pin Load Cell for Measuring of Contact Pressures in Extrusion and Compression Processes, Proceedings of 9th International Conference on Experimental Mechanics, Vol. 1, pp. 31-38, Copenhagen, Denmark, 1990.

[4] Osman F. H., Potel F., Loveday M. S.: Thtree-Dimensions Analysis of Contact Stress Measurements in Forming Processes, Proceedings, pp. 391-398, 6th ICTP, Erlanger – Nuremberg, 1999.

[5] Male A.T., Cockcroft M.G.: A Method for the Determination of the Coefficient of Friction of Metals under Conditions of Bulk Plastic Deformation, J. Inst. Metals, 1964-65, Vol.93, pp. 38-46.

[6] Čupković Đ., Plančak M., Vilotić D.: Influence of Initial Disc Dimensions on Diagram for Determination of Coefficient of Friction in Metal Forming (in Serbian), DEMI – 2002, International Conference, Republic of Serbska, Banja Luka, 2002.

[7] Garmong G., Paton N.E., Chesnutt J.C., Nevarez L.F.: An Evaluation of the Ring Test for Strain-Rate-Sensitive Materials, Metallurg. Trans., 1977, Vol.8A, pp.2026-2027.

[8] Dutton R.E., Goetz R.L., Shamasundar S., Semiatin S.L.: The Ring Test for P/M Materials, Trans. ASME J. Manuf. Sci. Engng, 1998, Vol.120, pp.764-769.