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ANALYSIS OF SURFACE PARAMETERS OF HELICAL GEARS WITH SMALLER NUMBER OF TEETH

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Abstract
In the paper, the condition and the causes of the failure modes of cylindrical gears are defined in selection to the most important running surface parameters of contacting teeth. Included are Hertz contact stress, minimum film thickness, tooth temperature rise. An improved analytical model is used by considering the load sharing ratio and specific kinematics of gear pairs. The aim of this paper is to accurately define the condition and causes of the failure modes to be able to determine corrective actions for analytical methods used in the design of cylindrical gear pairs. A parametric study is conducted for helical gear pairs with smaller number of pinion teeth.

Keywords: *helical gear, load sharing, film thickness, tooth temperature rise.*

1. Introduction

The cylindrical gear pairs with parallel shafts and involute profile can realize high values of the gear ratio by decreasing the number of pinion teeth. The experimental investigations indicated that the major failure modes of these gear pairs were pitting and scoring [1]. The design procedure of these gears requires to establish the optimum combination of geometrical and kinematic characteristics which provide the best performance of the running surface parameters under input conditions.

This study is undertaken to investigate these failure modes in selection to the most important running surface parameters of contacting teeth. Included are Hertz contact stress, minimum film thickness, tooth temperature rise. A parametric study is conducted to establish the optimum amount of the addendum modification coefficients from the viewpoint of the tribological criterion. In addition, the paper also includes an analysis for determining tooth load sharing.

2. Kinematic considerations

The characteristic of helical gears is mainly involved in the inclination of the contact lines. Both, the position and the length of the contact line at time t have a significant influence on the amount of the mesh stiffness of gears. Figure 1 shows some of the relevant features of the meshing plane of action for a pair of helical gears. The meshing starts at the point A, passes through point E_t and finishes at point E_x . In the kinematic analysis it is useful to consider an equivalent line of action AE as shown in Figure 1, where t_z is the meshing time period of passing a transverse base pitch of helical gears.

The position of the line of contact is indicated with the coordinate X of the equivalent line of action

$$X = X_0 p_b \tag{1}$$

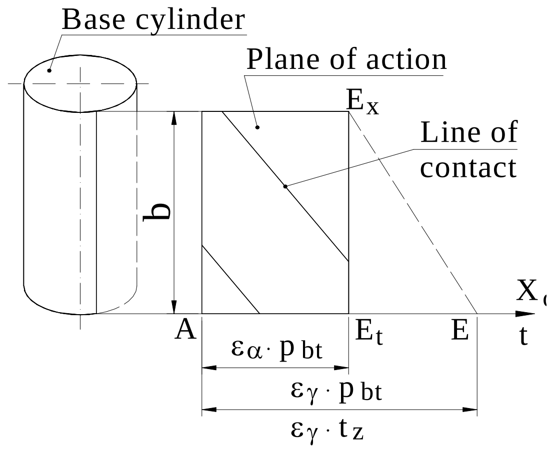


Figure 1: *Plane of action and the equivalent line of action*

where p_b is the transverse base pitch and $X_0 = 0 \sim \varepsilon_\gamma$.

At helical gears, the contact length of a tooth pair is not a constant during the engagement cycle. The time - varying of the length of a contact line is depended by the ratio between transverse contact ratio ε_α and overlap ratio ε_β. The instantaneous length of a contact line can be expressed as

$$l_c = \frac{\varepsilon_\alpha}{\varepsilon_\beta} \cdot \frac{b}{\cos\beta_b} \cdot c_x \tag{2}$$

where the parameter $c_x = f(\varepsilon_\alpha, \varepsilon_\beta)$ [3], b represents the face-width of the gear and β_b is the helix angle on the base cylinder.

3. Shared loads of meshing teeth

The transmitted load F_n is shared between the meshing teeth in the region of multiple pair contact. The condition of contact for each pair of teeth is expressed as

$$E = \frac{F_I}{K_I} + e_I = \cdots = \frac{F_i}{K_i} + e_i \tag{3}$$

and

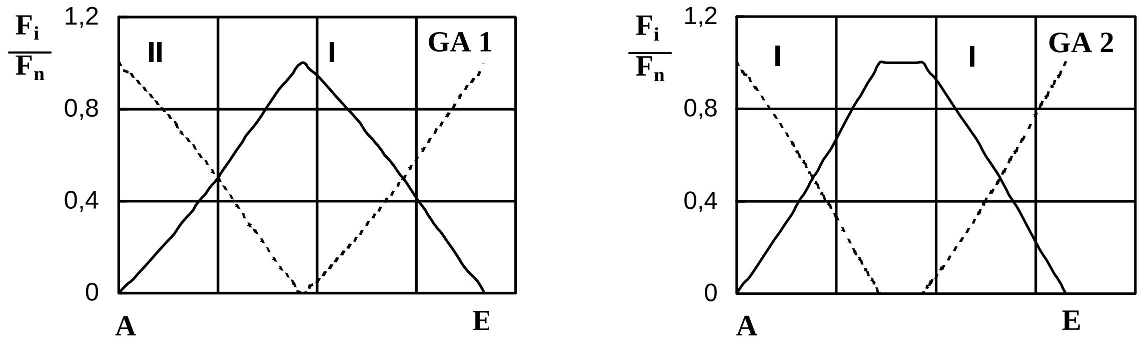
$$\sum_{i=I}^{N} F_i = F_n \quad \text{or} \quad \sum_{i=I}^{N} C_i = 1 \tag{4}$$

$$C_i = F_i / F_n \tag{5}$$

where F_i is the shared load, e_i is the composite error of i-th pair of teeth, and K_i represents the stiffness per unit length of a pair of gear teeth [3].

By solving Eqs. (3) and (4) for the case of double pair contact, the shared loads are expressed as

$$F_i = \frac{K_i F_n - K_i K_j (e_i - e_j)}{K_i + K_j} \tag{6}$$



(a) (b)

Figure 2: *Variation of load sharing ratio in meshing cycle*

where j = i+1. If j > II, than j = I. Equation (6) shows the effect of the mesh stiffness and tooth errors on the magnitude of the individual tooth load. The variation of the shared load factors of the gear pairs presented in Table 1 are shown in Figure 2 from the beginning to the end of contact, where I, and II indicate the two tooth pairs in contact.

3. Analytical Models of Tribological Parameters

It is known that the pitting is apt to occur first on the tooth surface where the contact pressure and relative negative sliding are the highest. This association is used as a criterion for design comparison.

3.1. Contact pressure

The contact pressure between the meshing teeth of the involute helical gears can be calculated by considering the contact between two truncated cones and is expressed as

$$p_H = Z_E \sqrt{W_n \left(\frac{1}{\rho_{n1}} + \frac{1}{\rho_{n2}} \right)} \qquad (7)$$

where

Z_E represents the elasticity factor,

W_n is the normal load per unit lenght, N/m;

ρ - radius of curvature at the point of contact, m.

The subscripts 1 and 2 denote the pinion and gear, respectively; the subscript n refers to normal plane.

Eq. (7) is valid at any point during the contact cycle and the contact pressure can be computed if the values of ρ_{n1} and ρ_{n2} are known.

3.2. Film Thickness

The prediction of the minimum film thickness in cylindrical gears is based on the analytical models of the elastohydrodynamic lubrication of line contacts [5], [6], [10]. In the actual analysis, the following models are considered:

a) The minimum film thickness in the isothermal elastohydrodynamic lubrication of rolling line contacts [6] can be written as

$$h_{o,I} = 3.07 \left(\frac{\eta_o u}{E \cdot \rho} \right)^{0.71} \left(\frac{W_n}{E \cdot \rho} \right)^{-0.11} (\alpha \cdot E)^{0.57} \qquad (8)$$

where:

η_o = viscosity at ambient pressure, Ns/m^2;

ρ = effective curvature radius, m;

α = pressure–viscosity coefficient of lubricant, m^2/N;

E = effective modulus of elasticity, N/m^2;

W_n = normal load per unit length, N/m;

u = surface velocity in direction of motion, m/s.

The effect due to the thermal effects is given [6] by the formula

$$h_o = h_{o,I} \left(1 + Q^{0.4} / 10 \right) \qquad (9)$$

where

$$Q = \frac{\gamma \eta_o u^2}{k_l} \qquad (10)$$

where:

γ = temperature – viscosity of lubricant, 1/°C;

k_l = thermal conductivity of lubricant, W/(m·°C).

b) The lubricant film thickness between gear teeth including both sliding and rolling motion of contacting surfaces [4] can be expressed as

$$h_o = c \left(\frac{\eta u}{2 W_n} \right)^{0.7} \left(\frac{W_n \cdot \alpha}{\rho} \right)^{0.6} \left(\frac{k_l}{\gamma \eta u_s^2 P_e^{0.5}} \right)^{d} \qquad (11)$$

where: u_s = sliding velocity, m/s;

γ = temperature – viscosity of lubricant, 1/°C; k_l = thermal conductivity of lubricant, W/(m·°C); $c = 0.5 \div 3$; $d = 0.3 \div 0.6$.

In Eq. (11), the following pressure – temperature viscosity relation is used

$$\eta = \eta_o e^{\alpha \cdot p - \gamma (T - T_o)} \qquad (12)$$

where: T = temperature, °C;

T_o = inlet lubricant temperature, °C;

η = viscosity of lubricant, Ns/m^2 ;

η_o = viscosity at ambient pressure, Ns/m^2.

A criterion used to determine the possibility of surface distress is the specific film thickness λ. When λ is less than 0.7 boundary lubrication prevails and lubricant, surface physical and chemical interaction, loads and temperature said to have a strong effect on distress modes and rates.

3.3. Surface Temperature Rise

Blok's theory states that scoring will occur when the maximum temperature T_c of contacting teeth exceeds a critical temperature T_{crit} , where

$$T_c = T_b + \Delta T \tag{15}$$

In eq. (15), T_b is the temperature of the tooth surfaces before meshing. The surface temperature rise ΔT in the thermal elastohydrodynamic lubrication of line contacts (Taylor, 1985) can be expressed as

$$\Delta T_1 = \frac{2\mu \cdot W_n \cdot u_s}{\left(\pi \cdot v_1 \cdot K_s \cdot \rho_s \cdot c_s \cdot b_H\right)^{0.5}} \psi_P \tag{16}$$

$$\Delta T_2 = \frac{2\mu \cdot W_n \cdot u_s}{\left(\pi \cdot v_2 \cdot K_s \cdot \rho_s \cdot c_s \cdot b_H\right)^{0.5}} \psi_G \tag{17}$$

where : ΔT_1 and ΔT_2 represent the temperature rise of the pinion and gear, respectively;

K_s = thermal conductivity of solid, $W/m \cdot ^\circ C$;

c_s = specific heat of steel gears, $J/kg \cdot ^\circ C$;

ρ_s = density of steel gears, kg/m^3;

b_H =semiwidth of contact;

v = instantaneous tangential velocity at contact point;

ψ = heat partition coefficients for pinion and gear.

It can be seen from Eqs. (16) and (17) that the temperature rise is directly proportional to the coefficient of friction and a reasonable evaluation of its value is essential for the temperature prediction. The instantaneous coefficient of friction is expressed as

$$\mu = 3\mu_m \frac{y}{l_{a,g}} - 1.5\mu_m \left(\frac{y}{l_{a,g}}\right)^2 \tag{18}$$

where

$$\mu_m = 0.12 \left(\frac{W_t R_{ao}}{u\eta_o}\right)^{0.25} X_R X\eta \tag{19}$$

where: X_R = roughness factor,

$X\eta$ = lubrication factor ;

W_t = tangential force per unit length, N/m;

R_{ao} = nominal surface roughness.

In Eq. (18), y is the distance between the contact point and pitch point and $l_{a,g}$ represents the length of approach path or recess path, respectively.

The computational methodology for the instantaneous coefficient of friction is based on the predicted mean values at the middle length of the approach and recess path [2]. The scoring is initiated when a local instantaneous total contact temperature in one point on the tooth flank exceeds a critical temperature T_{crit}. When this temperature is larger than 135^oC, the scoring probability is higher. If any of the above reach critical proportion scoring may occur in a sense of incipient scoring.

4. Results and discussion

Specifications of the analyzed gear pairs are shown in Table 1, where:

z represents the number of teeth,

m_n = normal module,

x_n = addendum modification coefficient (related to m_n) ,

x_{nmin} = minimum value of addendum modification coefficient to prevent undercutting; a_w = center distance, b = face-width; β = helix angle on the pitch cylinder;

l_g = length of action of approach;

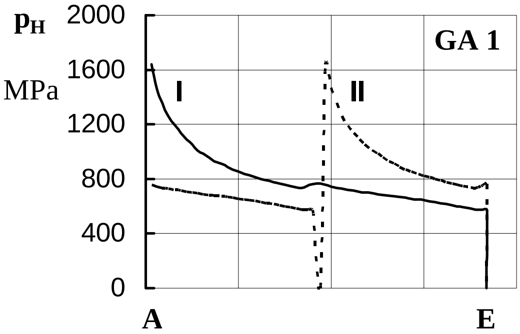
ε_α = transverse contact ratio,

ε_γ = total contact ratio.

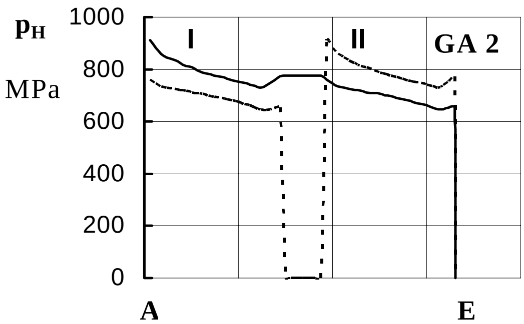
Test gears are made of 40Cr10 quenched and tempered steel. The tooth surfaces of these gears were finishing by hobbing. The initial surface roughness R_a was about 1.2 μm. The values of addendum modification coefficients are determinated from the geometrical conditions and the resistance of gear teeth to surface loading [1].

Table1. Specifications of the gear pairs

Gear Pairs	z_1	z_2	x_{n1}	L_g	ε_α	ε_γ
GA1	4	51	0.737*	2.23	0.89	1.98
GA2			1.100	-0.34	0.67	1.76
* x_{nmin}; m_n = 3 [mm]; a_w = 90 [mm]; b = 30 [mm]; β =20°						



(a)



(b)

Figure 3: *Variation of Hertz contact pressure along the equivalent line of action*

The results presented in Figs. 3 - 5 permit to analyse the variation of surface temperature rises, and calculated EHD film thickness in relation to maximum Hertzian stress. In these figures, A, E, C denote the beginning of engagement, end of engagement and pitch point of the pinion, respectively.

Figure 3 shows the distribution of the Hertz stress along the equivalent line of action. The contact Hertz pressure is found to be much improved as the addendum modification coefficient x_{n1} changes from 0.737 to 1.1.

The temperature rise on both the pinion and gear teeth contacting surfaces is plotted in Figure 4 as a function of the contacting position for two different speeds. The corresponding film thickness is shown in Figure 5 as a function of tooth contact position. These values are computed by using Eq. (11), where c = 2.06 and d = 0.54. The specific film λ is less than 0.7 and the boundary lubrication prevails in the case of helical gears with small number of pinion teeth.

It is seem that the temperature rise for both the pinion and gear is higher at the beginning than the ending of the engagement. The higher temperature rise and a reduced minimum film thickness result at the ending of the engagement when the pinion speed increases from n_1 = 1800 rpm to n_1= 3400 rpm.

The experimental results [1] evince that the contact fatigue life of test gears is governed by specific aspects of the pitting distributions on the teeth of the pinion and gear, in terms of addendum modification coefficients.

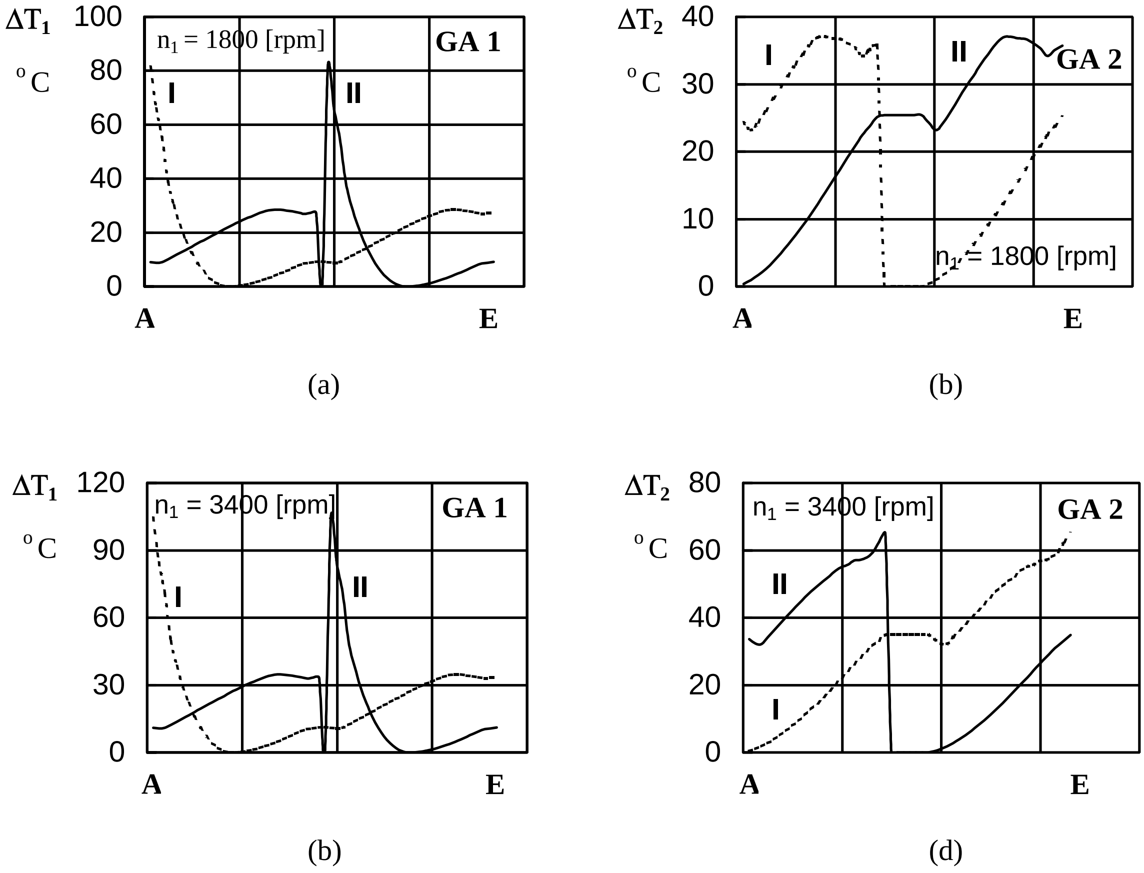


Figure 4: *Variation of temperature rise along the equivalent line of action*

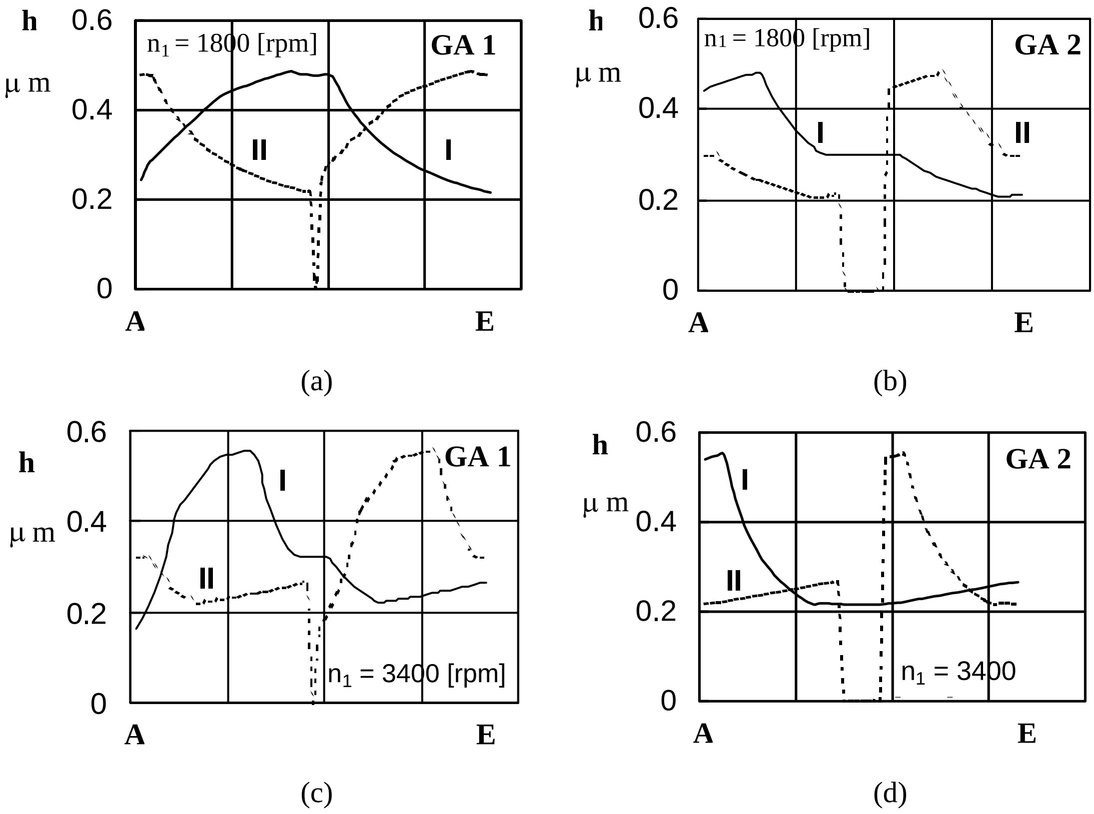


Figure 5: *Variation of film thickness along the equivalent line of action*

The experimental results [1] evince that the contact fatigue life of test gears is governed by specific aspects of the pitting distributions on the teeth of the pinion and gear, in terms of addendum modification coefficients. Thus, at the gear pair with $x_{n1} = x_{n1min}$ and $HB_1/HB_2 = 1$, the pitting cracks occurred near the tooth root of the pinion; at the mating gear the pitting occurred on the outside engagement zone. If the coefficient $x_{n1} = 1.1$, the pitting cracks occurred only on the teeth of the gear. In the mentioned situations, the pitting failure occurred on the zones with negative specific slidings. When $x_{n1} = 1.1$ and the ratio $HB_2/HB_1>1$, the pitting cracks may occur only on the tooth surfaces of the pinion. Under a heavy load, the surface temperature rise of the contact zone is high and the EHD film thickness decreases.

When the rotational speed of the pinion is larger than 1800 rpm, it seems that that the probability of metallic contact becomes high. An intermittent incipient scoring occurred on the tooth surfaces of the pinion with $n_1 = 3400$ rpm and $HB_1 = HB_2 = 3100$. This phenomenon may occur when the load exceeds a critical limit value. Therefore, the initial tooth surface should be finished as smoothly as possible in order to obtain high surface durability of these helical gears.

5. Conclusions

Analyses of the most important parameters, which are believed to act on the tooth surfaces and are significant among influences on the failure modes are described. The prognosis of the running surface parameters is improved by using the shared load in the analytical model of the tribological parameters. The load sharing ratio is analytically expressed by considering the mesh stiffness of each individual tooth pair that is simultaneously in contact.

The best results of the surface durability were obtained when the amount of the addendum modification coefficient of the pinion was a little large than the limit of the undercut and a low hardness pinion was combined with a harder gear.

The cylindrical gears with high gear ratio and smaller number of pinion teeth operate under conditions of boundary lubrication. The probability of scoring failure is higher in the case of larger values of the pinion speed.

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