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DEVELOPMENT OF AN ALGORITHM AND PROGRAM SYSTEM TO STABILITY PROBLEM OF HD JOURNAL BEARINGS

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Abstract

An original algorithm for solving the stability problem of the dynamic system "lubricant film – shaft" in HD journal bearing is suggested. The finite length lubricates with Newtonian fluid in a laminar regime, such the convective inertia forces of the lubricant is rendered an account. The rigid rotor is well balanced, such is investigated the motion of the shaft centre, provoked from the unbalanced HD forces of the lubricants.
The algorithm is based on the modified stability criteria by Hurwitz and Ljapunov.
For automating of investigation a new program system, which is adequate of the theoretical treatment is created. The system is reduced in a form, friendly for use and by users, unacquainted in details with the serious mathematical apparatus. The program is adaptive and can be modifying for diverse kind of related HD problems, i.e. non-Newtonian lubricants, deformability of the bearing surfaces (EHD problems), etc.
The program system is intended for scientific investigation, but can be used and in direct engineer practice.

Key words: *Hydrodynamic (HD) lubrication, convective fluid inertia, stability of journal bearing*

1. INTRODUCTION

Journal bearings are used extensively in rotating machines because of their low wear and good damping characteristics. Typical applications include turbines, large milling systems, engine cranks, compressors, gearboxes, etc. In these bearings, a hydrodynamic film occurs when there is sufficient lubricant between the lubricated surfaces at the point of loading to form a fluid wedge that separate the sliding surfaces.
Notwithstanding intensive scientific work in the field of journal bearings lubrication, many tribologists are confining in the region of steady state and did not consider the dynamic stability characteristics of this significant tribological system.
As one knows, the steady state characteristics can be considered when designing a journal bearing. It is observed, however, that "oil whirl" phenomena [1] are encountered in rotating journal bearing. The whirl frequency is set by speed at which the shaft can pump the fluid around in the clearance and maintain the pressure pattern, which produces the driving force. Meanwhile, the frequency of the self-excited vibration is equal to half of the angular velocity, the so-called "half frequency whirl". The phenomena of self-excited vibration have important effects on the performance characteristics of the system. If the amplitude of oil whirl oscillations becomes too large, the radius of the boundary cycle is larger than the radial clearance of the bearing, and thus, this oil whirl vibration leads to the journal-bush

contact, which can lead to unsafe operation.
This problem necessitates investigating the stability of the equilibrium position of the shaft in journal bearing film, which is the object of many old and recent papers.
The determination of the stability threshold speed of a journal bearing can be measured from the stiffness and damping coefficients [2]. The classical method to evaluate the dynamic bearing coefficients is based on a numerical differentiation of the Reynolds equation. Alternatively [3], before the numerical evaluation is performed, one can doing the mathematical differentiation of Reynolds equation and, the dynamic coefficients are found directly by solving the equations. Moreover, the reckoning of dynamic coefficients is important, since they can be used to depict the film forces caused by small amplitude disturbance of journal bearing about its steady equilibrium position. The eight oil-film coefficients are used to determine the stability behaviour of rotors and by the Routh-Hurwitz stability criterion the stability threshold speed of a rotor bearing is investigated under various eccentricity ratios. For small amplitude motion neighbouring the equilibrium solution, the first order perturbation method is suitable and often used. Studies related with the similar stability analysis with determination namely of above mentioned coefficients are abundant [4, 5, 6, 7, etc.]
The other groups of investigators use so-called stability parameter for critical stability determination. This parameter is a function on relative eccentricity, bearing geometrical proportions and operational parameters. In this case verification for stability of the system works up after determination of basic bearing characteristics. The similar solutions are based basically on the Korovchinskii [8] and Poznjak [9] methods to receive the critical value of stability parameter. For that purpose must be solve perturbation differential equation of hydrodynamic. In this case the test for dynamic system stability carries out by Hurwitz; such very complicated mathematical transformations are used. The grave calculating work is not program provided (or an access to similar program systems is missing). Such for solution of this kind of problems, the specialists (scientific workers and engineers) must be master abovementioned complicated method.
In an earlier our study [10] a new solution of the stability problem was presented and modified stability criteria of the considered tribological system was derived. The elaborated and demonstrated there method of approach is easier for use and along with that is program provided with a suitable program system. But the suggested algorithm and program system are concerned only to the classical lubrication theory case.
To meet the specific requirements of industries to modern machine elements, bearings must be work successfully under wide range of speeds. But in bearings operating under a high speed and using low-viscosity oils the fluid inertia forces cannot be neglected even the flow is laminar. It is well known that the classical Reynolds equation is not valid in this situation.
With reference to all mentioned, the object of the present paper is to development of an algorithm and program system for solving of HD journal bearings stability problem with consideration of convective fluid inertia of the lubricant.
Note: All used notations and symbols are the traditional in classical HD theory of lubrication and in vibration theory. By this reason a separate nomenclature is not presented here.

2. THEORETICAL BACKGROUND

2.1. Hydrodynamic equations and steady state characteristics

The modified Reynolds equation [11] governs the pressure distribution in the fluid film of a hydrodynamic journal bearing:

$$\left[1-\frac{Re^*}{4}\frac{\partial H}{\partial \theta}H\right]\frac{\partial^2 \Pi}{\partial \theta^2}+\alpha\left[1-\frac{Re^*}{12}\frac{\partial H}{\partial \theta}H\right]\frac{\partial^2 \Pi}{\partial {z_1}^2}+$$
$$+\left[\frac{3}{H}\frac{\partial H}{\partial \theta}-\frac{Re^*}{24}\left(\frac{\partial^2 H}{\partial \theta^2}H-4\left(\frac{\partial H}{\partial \theta}\right)^2\right)\right]\frac{\partial \Pi}{\partial \theta}+$$
$$+\alpha\left[\frac{3}{H}\frac{\partial H}{\partial z_1}-\frac{Re^*}{12}\left(\frac{\partial^2 H}{\partial \theta \partial z_1}H-4\frac{\partial H}{\partial \theta}\frac{\partial H}{\partial z_1}\right)\right]\frac{\partial \Pi}{\partial z_1}-$$
$$-\frac{5}{24}\alpha\frac{\partial^2 \Pi}{\partial \theta \partial z_1}\frac{\partial H}{\partial z_1}H=$$
$$=\frac{1}{H^3}\frac{\partial H}{\partial \theta}-\frac{Re^*}{36}\left[\frac{1}{H}\frac{\partial^2 H}{\partial \theta^2}+\frac{2}{H^2}\left(\frac{\partial H}{\partial \theta}\right)^2\right]. \quad (1)$$

All of the terms of above equation, in which the generalized Reynolds number Re^* participates as a coefficient, take into account the contribution of convective inertia forces to the hydrodynamic pressure gradient.
The film thickness for rigid bearing surfaces can be expressed by

$$H(\theta, z_1) = 1 + \varepsilon\cos\theta\,. \quad (2)$$

The resultant dimensionless load carrying capacity $\bar{W}$ can be determined as follows

$$\bar{W} = \sqrt{\bar{W}_x^2 + \bar{W}_y^2} = \frac{\beta^2}{6\eta\omega r L} W\,, \quad (3)$$

where

$$\bar{W}_1 = -\int_{-1}^{1}\int_{0}^{2\pi} \Pi\cos\theta\, d\theta dz_1\,, \bar{W}_2 = \int_{-1}^{1}\int_{0}^{2\pi} \Pi\sin\theta\, d\theta dz_1\,. \quad (4)$$

As a result the steady attitude angle γ and Sommerfeld number S are calculated respectively by

$$\gamma = \operatorname{arc}\tan\frac{\bar{W}_2}{\bar{W}_1} \quad (5)$$

$$S = \frac{W\beta^2}{\eta\omega r L} = 6\bar{W}\,. \quad (6)$$

2.2. Stability problem exposition and modified stability criteria

It is assumed, that a rigid rotor is good balanced, so that the centrifugal inertia forces of unbalanced masses are excluded from the shaft centre equations of motion. They may be expelling even by reasons, that here will investigate the stability zone of the lubricant film.

The task is formulated by the following way: at a specified behaviour, the shaft centre position is determined by the relatively eccentricity ratio, which corresponds to a point from the dynamic equilibrium curve.

Must defining the character of the shaft center motion, if this center brings out from dynamic equilibrium position and moves in other point. It is well known, that in this case exist two possibilities: damping oscillations or instable motion with rising amplitude values, which leads to breakdown situations.

In the present investigation is considered the motion of the shaft center, provoked from the unbalanced hydrodynamic forces of the lubricant. The shaft supports a constant load W in the equilibrium position. The equilibrium position is dynamic and it is determined by co-ordinates eccentricity ratio ε and attitude angle γ. It is assumed, that the troubled motion is oscillations with small amplitude values, which enables to make a linearization.

To receiving the modified stability criteria of the dynamic system is necessary to solve the differential equations of motion of the shaft centre. When the small plane oscillations about a position of equilibrium, which are correspond to the own weight of the rotor, are considered in the relevant differential equations are involve the components of the HD forces and these of the tensor of damping forces [12, 13, 9].

Through roots values of this equation can be draw up the Hurwitz matrix, whose principal minors must be positive [14].

For the considered case, the Hurwitz conditions are transforming to the following expresses [10]:

$$\bar{\Delta}_1 = \alpha_1 > 0\,;\ \bar{\Delta}_2 = \alpha_1\alpha_2 - \alpha_3 > 0\,;$$
$$\bar{\Delta}_3 = \alpha_3\bar{\Delta}_2 - \alpha_1^2\alpha_4 > 0\,;\ \bar{\Delta}_4 = \alpha_4\bar{\Delta}_3 > 0\,. \quad (7)$$

where

$$\alpha_1 = 2\left(B_{xx} + B_{yy}\right); \quad (8)$$
$$\alpha_2 = T_{11} + T_{22} + 4\left(B_{xx}B_{yy} - B_{xy}^2\right);$$
$$\alpha_3 = 2\left(B_{yy}T_{11} + B_{xx}T_{22}\right) - 2B_{xy}\left(T_{12} + T_{21}\right);$$
$$\alpha_4 = T_{11}T_{22} - T_{12}T_{21}\,.$$

In the above expresses introduced symbols can be read as follows:

$$B_{xx} = \varepsilon^2 B_{\eta\eta} + 2\varepsilon\sqrt{1-\varepsilon^2}B_{\eta\xi} + \left(1-\varepsilon^2\right)B_{\xi\xi}\,; \quad (9)$$
$$B_{xy} = B_{yx} = \varepsilon\sqrt{1-\varepsilon^2}B_{\eta\eta} + \left(1-2\varepsilon^2\right)B_{\eta\xi} - \varepsilon\sqrt{1-\varepsilon^2}B_{\xi\xi}\,;$$
$$B_{yy} = \left(1-\varepsilon^2\right)B_{\eta\eta} - 2\varepsilon\sqrt{1-\varepsilon^2}B_{\xi\eta} + \varepsilon^2 B_{\xi\xi}\,;$$

$$B_{\eta\eta} = \frac{3\pi\sqrt{g/c}}{4\omega\left(1-\varepsilon^2\right)^{3/2}\left[1 + 1{,}5\alpha^2\dfrac{1-\varepsilon^2}{1+\varepsilon^2}\right]}\frac{1}{S}\,; \quad (10)$$

$$B_{\eta\xi} = B_{\xi\eta} = \frac{1}{2\omega}\sqrt{\frac{g}{c}}\,;$$

$$B_{\xi\xi} = \frac{1}{2\omega}\sqrt{\frac{g}{c}}\frac{\sqrt{1-\varepsilon^2}}{\varepsilon}\,.$$

$$T_{11} = 1 + \frac{1}{\sqrt{1-\varepsilon^2}}\,;\ T_{22} = \frac{1}{S}\varepsilon\frac{dS}{d\varepsilon}\,; \quad (11)$$

$$T_{12} = \frac{\varepsilon}{1-\varepsilon^2} - \frac{\sqrt{1-\varepsilon^2}}{\varepsilon}\,;\ T_{21} = \frac{1}{S}\sqrt{1-\varepsilon^2}\frac{dS}{d\varepsilon}\,.$$

Here to evaluate the derivation of Sommerfeld number S is used the express, given by Kodnir and improved by us

$$S = \frac{W\beta^2}{\eta\omega r L} = \left(1 + 0{,}3\varepsilon\right)\times \quad (12)$$
$$\times\frac{1{,}02\left[1-\left(1-\varepsilon\right)^4\right]}{\left(1-\varepsilon\right)\left\{1 + \alpha^2\left[0{,}12 + 2{,}31\left(1-\varepsilon\right)\right]\right\}}\,.$$

A full determination of modified stability criteria are detailed presented in [10].

3. SIMULATION PROCEDURE - ALGORITHM AND PROGRAM SYSTEM

To solving the stability problem of considered tribological system an original program system HDSTABJB has been created. HDSTABJB is intended to verification by introduced criteria of system stability in finite journal bearing with consideration of convective fluid inertia terms. It is based on the command methods of the mathematical analysis and programming theory. The program system is reduced in a form, which is friendly for use and by users, unacquainted in details with the serious mathematical apparatus. For a normal function on a PC are necessary common hardware and processor time resources. The system has two basic sections. First of them is concern to the hydrodynamic part of the problem and second to the stability study. The block-scheme of the algorithm is given on a Fig. 1.

● To carrying out of the numerical experiments the program system is organized in interactive mode, such the basic operational parameters are appeared in a detached file-window (INP.DAT). The constructive bearing's and lubricant's parameters, which may to vary free are: shaft radius r, [m]; bearing length L, [m] and/or diameter to length ratio α ($\alpha = 2r/L$); bearing clearance c, [m] and/or relative clearance β ($\beta = c/r$); eccentricity ratio ε; dynamic viscosity of the lubricant η, [Pa.s]; revolutions per minute n, [r.p.m.] and/or angular velocity ω, [rad/s]; generalized Reynolds number Re* ($\mathrm{Re}^* = \beta \mathrm{Re}$). The use of similar separated window gives opportunity for great number of calculating results at easy change of the abovementioned quantities.

● To receive the steady state performance characteristics must be solve the dimensionless modified Reynolds equation (1). This equation is solved numerically using the finite difference method. The film domain is divided by the grid spacing. The mesh has ($m-1$) intervals in the circumferential direction and ($n-1$) intervals across the bearing length. Consequently each intersection of these dividing lines is to give a mesh size of ($m \times n$) points. Different mesh sizes have been tried and a mesh with 66 intervals in the circumferential direction and 12 intervals across the bearing width is used. This size gives a rapid rate of convergence and

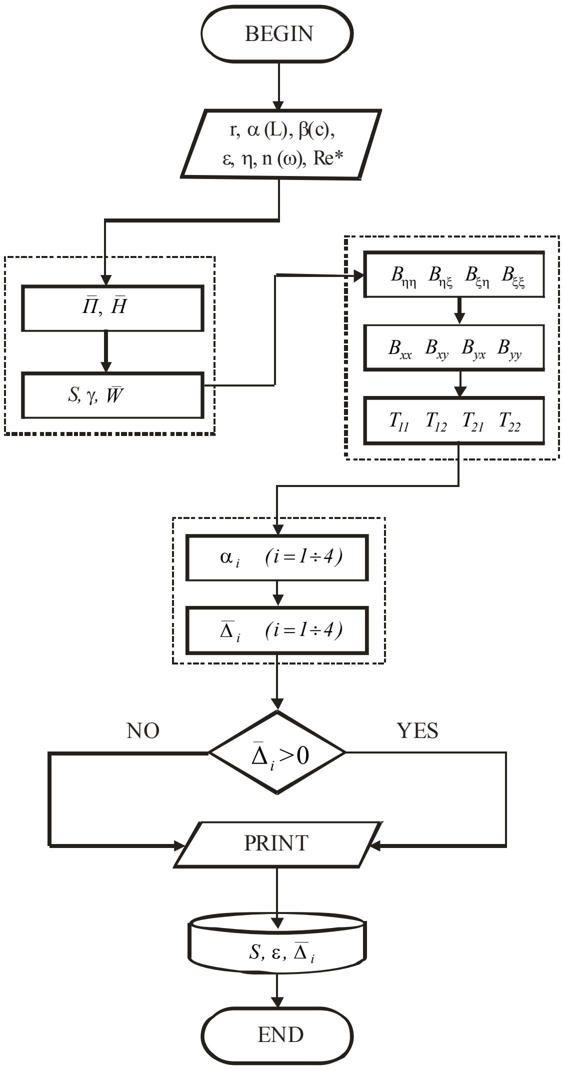

Fig. 1: *Block-scheme*

agreeable computer working time. Over-relaxation method is used in order to improve the convergence rate [5, 12, etc.], as the iterative procedure terminated when the difference in the successive iterations becomes less than the predefined tolerance

$$\delta = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\left|\Pi_{i,j}^{(k)} - \Pi_{i,j}^{(k-1)}\right|}{\sum_{i=1}^{m}\sum_{j=1}^{n}\Pi_{i,j}^{(k)}} \le 1.10^{-7}. \tag{13}$$

For steady state pressure distribution the Reynolds boundary conditions is used, namely:

$$\Pi\big|_{\theta=\theta_1} = \Pi\big|_{\theta=\theta_2} = 0; \qquad \left.\frac{\partial \Pi}{\partial \theta}\right|_{\theta=\theta_2} = 0;$$

$$\Pi\big|_{Z_1=\pm1} = 0; \qquad \left.\frac{\partial \Pi}{\partial z_1}\right|_{Z_1=0} = 0, \tag{14}$$

where θ_1 and θ_2 are the angle coordinates at which the film commences and cavitates respectively. Since the bearing is symmetrical about its central plane ($z_1 = 0$), only one-half of the bearing needs to be considered for the analysis.

HD pressure $\bar{\Pi}$ and film thickness $\bar{H}$ distribution, as well as steady state performance parameters $S, \gamma, \bar{W}$ are obtained by program code CONVWIN, which is elaborated on BORLAND DELPHI ENTERPRISE 7. This program is possessed of convenient interface, as the output results (OUT.DAT) are arranged in a suitable structure to integration in a following part of the program.

● For the calculation of stability branch of the problem, the program module STAJB has been created. STAJB is developed in MATCAD 2000 PROFESSIONAL environment, such it is based on the adequate to the theoretical treatment in paragraph **2.2** algorithm. In the beginning, at use of the initial data (especially ε) and calculated values of the Sommerfeld number S and load carrying capacity $\bar{W}$ (corresponding load W), must be calculate the following dimensionless values [10]: B_{ij} in $O\eta\xi$ coordinate system; B_{ij} in Oxy coordinate system; T_{11}, T_{12}, T_{21}, T_{22}. Then the program carries out the computation of the non-dimensional values $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ of the characteristic equation coefficients, which is a polynomial of the 4th order. Subsequently these values are used to evaluate the non-dimensional principal minors $\bar{\Delta}_1, \bar{\Delta}_2$, $\bar{\Delta}_3, \bar{\Delta}_4$. After that it is possible to make the test for stability of the dynamic system, i.e. consecutively satisfaction of Hurwitz's criteria, which is given with inequalities (7). The system is considered to be stable if all of these minors are positive. If any of them is negative, the system becomes unstable.

4. APPLICATION EXAMPLES - NUMERICAL RESULTS

The numerical results, which are presented in Tables 1-3, are obtained at the following parameters

$r = 15.10^{-2}$ [m]; $c = 3.10^{-4}$ [m];

$\eta = 0,04$ [Pa.s]; n = 3000 [r.p.m].

For all considerate cases the verification for the stability of system is made by introduced criteria.

Table 1.

$\alpha = 0,5$	$Re^* = 0$	$Re^* = 1,8$	stability
ε	S	S	
0,1	0,189	0,191	No
0,2	0,400	0,403	No
0,3	0,655	0,661	No
0,4	0,993	1,003	No
0,5	1,49	1,511	Yes
0,6	2,309	2,355	Yes
0,7	3,912	4,023	Yes
0,8	8,119	8,436	Yes
0,9	27,289	28,709	Yes

Table 2.

$\alpha = 2$	$Re^* = 0$	$Re^* = 1,8$	stability
ε	S	S	
0,1	0,064	0,064	No
0,2	0,136	0,138	No
0,3	0,227	0,233	No
0,4	0,354	0,364	No
0,5	0,547	0,563	No
0,6	0,879	0,905	Yes
0,7	1,561	1,606	Yes
0,8	3,455	3,546	Yes
0,9	12,585	12,897	Yes

Table 3.

$\alpha = 1$	$Re^* = 0$	stability	$Re^* = 1,8$	stability
ε	S		S	
0,525	1,043	No	1,067	No
0,53	1,066	No	1,091	No
0,535	1,09	No	1,116	No
0,54	1,115	No	1,142	Yes
0,545	1,14	Yes	1,168	Yes
0,55	1,167	Yes	1,195	Yes
0,555	1,197	Yes	1,222	Yes

From the results is clear that at lower values of the eccentricity ratio the system is in an unstable zone. Along with that it is evident that in the case with consideration of lubricants convective inertia terms the Sommerfeld number is higher, which leads to change of the critical stability of the journal bearing.

5. CONCLUSIONS

An original algorithm for solving the stability problem of the dynamic system "lubricant film – shaft" in HD journal bearing is suggested. The finite length lubricates with isoviscous Newtonian fluid in a laminar regime the lubricants inertia forces are taken into consideration. The rigid rotor is good balanced, such is investigated the motion of the shaft centre, provoked from the unbalanced HD forces of the lubricant.

The algorithm is based on the modified stability criteria by Hurwitz and Ljapunov, such the elaborated method of approach is easier for use than known theoretical methods.

For automation of investigation a program system, which is adequate of the theoretical treatment is created. The system is reduced in a form, friendly for use and by users, unacquainted in details with the serious mathematical apparatus. The new-created program HDSTABJB is powerful and it is not memory aggressive, such for the normal function on a PC is necessary common hardware and processor time resources. It isn't requiring a special preliminary preparation of working station, i.e. additional computer related systems, devices and peripherals. The current system is appropriate for independent use, but it is also possible incorporate to various CAM / CAD systems. The program system is developed in the surrounding of widespread and accessible mathematical package – MATCAD 2000 PROFESSIONAL. The system is adaptive and can be modifying for diverse kind of HD problem's treatment i.e. non-Newtonian lubricants, deformability of the bearing surfaces (EHD problems), etc. The system is intended for scientific investigation, but can be used and in direct engineer practice (design phase and/or operating mode of the bearings system).

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