

**BALKANTRIB'05**  
**5<sup>th</sup> INTERNATIONAL CONFERENCE ON TRIBOLOGY**  
**JUNE.15-18. 2005**  
**Kragujevac, Serbia and Montenegro**

---

**THE DETERMINATION OF THE TEMPERATURE FIELD IN  
THE LUBRICANT FILM OF HYDRODYNAMIC  
JOURNAL BEARINGS**

*Mihail Ionescu*  
*“Stefan cel Mare” University, Suceava, Romania*

**Abstract**

*The functional parameters of hydrodynamic bearings imply rigorous knowledge of the thermal condition in the lubricant film. The importance of this type of knowledge has been borne out throughout the ages by countless experimental and theoretical researches. The present paper proposes an analytical solution of the thermal balance equation, as well as its validation. In this respect, a detailed analysis is made regarding the behaviour of the mathematical calculation model by comparison with the experimental data furnished by the literature in the field in a number of concrete cases.*

**KEYWORDS:** *thermohydrodynamic, analytical solution, journal bearing, film, temperature field*

**1. INTRODUCTION**

Modern installations, devices and machines frequently manifest, embedded in their structures, constructive solutions which make use of hydrodynamic radial sliding bearings. Owing to the forces of friction among the layers of lubricant film, as well as between the film and the metallic parts of the bearing, a great amount of the hydraulic energy turns into heat. The temperature variation in the lubricant film and its consequences on viscosity have been thoroughly researched over the years.

At present, the solution of the thermal balance equation is performed numerically. In this respect, mention must be made of the research and methods proposed [1-3]. The numerical methods are laborious, time-consuming and, with the exception of the THD solution often inaccurate.

**2. NOMENCLATURE**

$C$  = radial clearance (m)  
 $c$  = lubricant specific heat ( $J / Kg^0 K$ )  
 $e$  = eccentricity (m)  
 $h$  = film thickness (m)

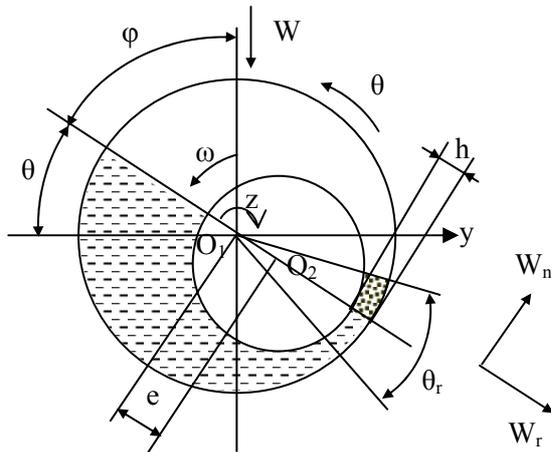
$h_m$  = maximum film thickness (m)  
 $h_0$  = film thickness, in which the pressure gradient is zero (m)  
 $I(\theta) = 1 - \frac{(L - L_1) \cdot e \cdot \sin \theta}{ho \cdot R + L \cdot e \cdot \sin \theta}$ , correction function introduced by Ionescu [4]  
 $L_1$  = unitary length,  $L_1 = 10^{-5}$ (m)  
 $L$  = real length bearing, (m)  
 $n = \rho \cdot c \cdot h^2 \cdot \tau_{1,2}$ , notation ( $J / m^0 K$ )  
 $p, p_a$  = pressure, supply pressure at inlet (Pa)  
 $Q_{rec}$  = recirculating flow rate ( $m^3/s$ )  
 $Q_l, Q_{lc}$  = leaking flow rate out of sides, rectified leaking flow rate out of sides ( $m^3/s$ )  
 $Q_h, Q_{h_m}$  = flow rate calculated for film thickness  $h$ , respectively  $h_m$  ( $m^3/s$ )  
 $q_x, q_z$  = unitary flows on x and z directions, respectively, ( $m^2/s$ )  
 $R$  = shaft radius (m)  
 $T_i$  = initial temperature ( $^0C$ )  
 $T_{mf}$  = medium temperature of the film ( $^0C$ )  
 $T_s$  = shaft temperature ( $^0C$ )

$V$  = runner velocity (m/s)  
 $W$  = load carrying capacity (N)  
 $x, y, z$  = global coordinates (m)  
 $\beta$  = temperature - viscosity coefficient ( $^{\circ}\text{K}$ )  
 $\varepsilon$  = eccentricity ratio  
 $\theta, \theta_i, \theta_r$  = circumferential angular coordinate, initial angles, and respectively rupture angle of the lubricant film (degree)  
 $\mu_1$  = inlet viscosity (Pa·s)  
 $\mu$  = lubricant viscosity (Pa·s)  
 $\rho$  = fluid density ( $\text{Kg/m}^3$ )  
 $\tau_{1,2}$  = represents the ratios  $\frac{Q_{lc}}{2Q_h}$ , respectively  
 $\frac{Q_l}{2Q_h}$   
 $\chi_y$  = thermal conductivity of the fluid ( $\text{W/m}^{\circ}\text{K}$ )

### 3. GOVERNING EQUATIONS

A thermohydrodynamic analysis (THD) of a journal bearing refers to a realistic solution of the generalized Reynolds equation in which the viscosity field is predicted based on the computation of temperature obtained from the conservation of energy.

To identify the denominations, the principle schema of the radial bearing of Figure 1 will be considered.



**Figure1:** Finite radial bearing

The starting point of the new model is the differential relation of pressures, proposed and thoroughly presented in paper [4]:

$$\frac{\partial}{\partial z} \left( \frac{h^3}{\mu} \cdot \frac{\partial p}{\partial z} \right) = 6 \cdot V \cdot \frac{\partial h}{\partial x} \cdot I(\theta) \quad (1)$$

By means of this equation, the film thickness  $h_0$ , the recirculating flow rate and the leaking flow rates out of sides are calculated.

To this effect, we proceed as follows:

a) the gradient in the movement direction is annulled and  $\theta_r$  is calculated, observing the boundary conditions  $\left( p = p_a, \frac{\partial p}{\partial x} = 0 \right)$ ;

b) then, we can calculate the film thickness  $h_0$  with the relation:

$$h_0 = c + e \cdot \cos(\theta_r) \quad (2)$$

c) the flows are calculated by means of the relations:

$$\begin{aligned}
 Q_{rec} &= \frac{VL}{2} h_0, \\
 Q_l &= \frac{VL^2 e}{2R} \int_{\theta_i}^{\pi} \sin(\theta) \frac{2h_0 R + L_1 e \sin(\theta)}{2h_0 R + L e \sin(\theta)} d\theta \quad (3) \\
 Q_{lc} &= Q_l + \frac{VL}{2} \int_{\theta_i}^{\pi} \left[ h_0 \left( I(\theta) - \frac{1}{\pi} \right) \right] d\theta
 \end{aligned}$$

d) If  $Q_{rec} + Q_l \geq Q_{h_m}$ , then  $Q_l$  will continue to be used; if  $Q_{rec} + Q_l < Q_{h_m}$ , then  $Q_{lc}$  will be used.

Then, the thermal balance equation proposed by Constantinescu [5] is considered.

$$\begin{aligned}
 \rho c \left[ q_x \frac{\partial T_m}{\partial x} + q_z \frac{\partial T_m}{\partial z} \right] &= \\
 &= \chi_y + \frac{\mu V^2}{h} + \frac{h^3}{12\mu} \left[ \left( \frac{\partial p}{\partial x} \right)^2 + \left( \frac{\partial p}{\partial z} \right)^2 \right] \quad (4)
 \end{aligned}$$

Equation (4) applies to the laminar flow regime. Account is taken of the fact that the temperature variation with respect to the direction of the shaft's movement is due to the pressure variation with respect to the same direction. In agreement with the latter statement, the gradients of medium temperature and of pressure in relation to movement direction are ignored in the balance equation. In this simplifying hypothesis equation (4) takes the form:

$$\rho c q_z \frac{\partial T_m}{\partial z} = \chi_y + \frac{\mu V^2}{h} + \frac{h^3}{12\mu} \left( \frac{\partial p}{\partial z} \right)^2 \quad (5)$$

In order to simplify calculations, an adiabatic evolution is considered in the lubricant film.

Depending on the discussion at step d, the film's medium temperature variation law is obtained analytic by means of the thermal balance equation:

$$T_{m_{1,2}} = \frac{1}{\beta} \ln \left[ \exp(\beta T_i) \frac{n + 2\mu_1 \beta V z (1 + 3\tau_{1,2}^2)}{n} \right] \quad (6)$$

The expression of the lateral flows at sides by means of the relations  $\tau_{1,2}$  affords an effective graphical representation of the lubricant film's temperature field.

Subsequently the lubricant film's medium temperature is calculated, and then its medium viscosity, by means of the equation:

$$\mu = \mu_1 \cdot e^{-\beta(T_s - T_i)} \quad (7)$$

The film's medium temperature is considered approximately equal to that of the shaft.

#### 4. VALIDATION

For the validation of the method, the temperature field and the medium field temperature are calculated for a number of concrete cases provided by the technical literature in the field. The concrete cases under analysis are those exemplified by Khonsari, [3] and involved in the experimental studies, [6-8].

**Table 1.**

Solution	THD solution		Proposed solution	
	$\varepsilon$	$T_{\text{shaft}} (^{\circ}\text{C})$	$\varepsilon$	$T_{\text{shaft}} (^{\circ}\text{C})$
Experimental data				
Dowson, W=11000N	0.574	47.74	0.579	47.43
Ferron, W=4000N	0.577	44.10	0.579	43.54
Ferron, W=6000N	0.543	47.06	0.536	46.87
Mitsui, W=3920N	0.452	49.88	0.431	48.06

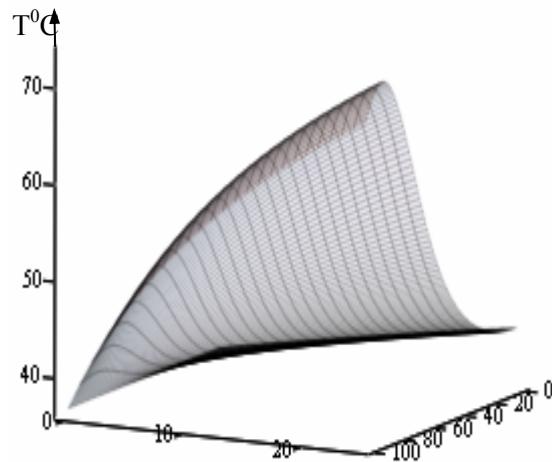
Table 1 presents the main parameters determined numerically by means of the THD solution and, respectively, obtained analytically by means of the proposed solution.

The differences between the proposed model and the THD solution increase with the reduction of eccentricity; this is due to the boundary condition used for the calculation of the film's rupture angle.

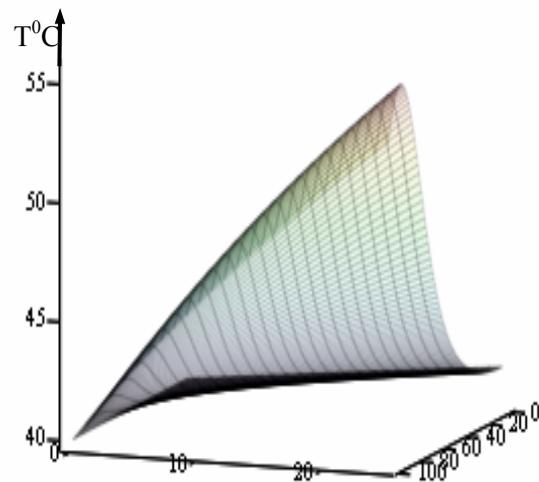
Thus, by comparison with the THD solution,

the eccentricity ratios and the film's medium temperatures calculated by means of the model proposed in the paper display errors in the range of (+ 0.3% ÷ - 4.8%) and (-0.4% ÷ -3.6%) respectively.

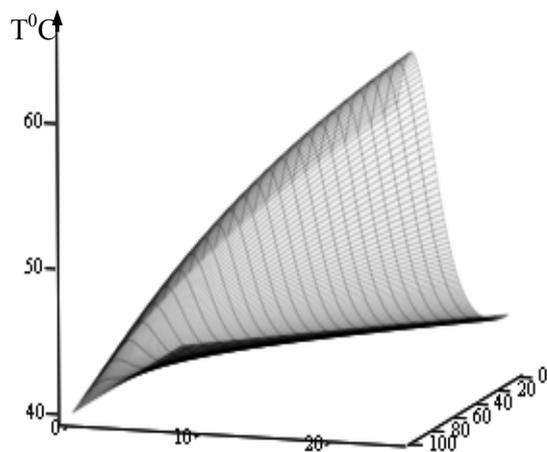
Figures 2–5 represent graphically the temperature fields corresponding to the four concrete situations under analysis. The expression of the leaking flow rate out of sides by means of equivalent flows according to film thickness  $h$  affords a satisfactory graphical representation. Nevertheless, the graph's aspect may be somewhat different from reality.



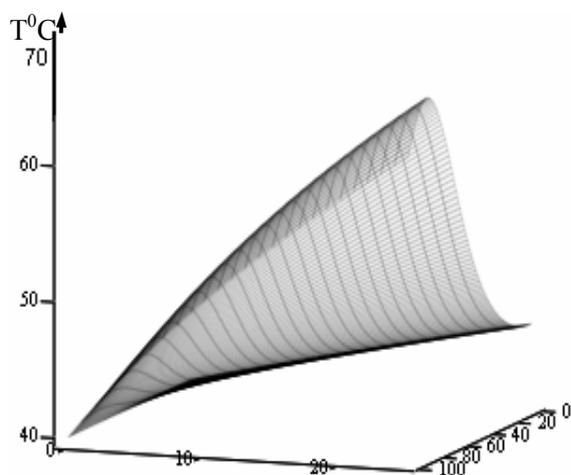
**Figure 2:** Film temperature distribution [Dowson , W=11000N]



**Figure 3:** Film temperature distribution [Ferron , W=4000N]



**Figure 4:** Film temperature distribution [Ferron , W=6000N]



**Figure 5:** Film temperature distribution [Mitsui , W=3920N]

## 5. CONCLUSIONS

Theoretical and experimental results are found in good agreement. The calculation accuracy is comparable to that offered by the THD and ISOADI methods.

The calculation times are very small compared to those required by the numerical methods for the solution of the thermal balance equation.

Owing to the form of the differential equation for pressures, the possible boundary conditions for the pressure field are those specific to large and medium load bearings. Consequently, the present model yields good results for such bearings. According to Constantinescu, [5],

condition  $\left( p = p_a, \frac{\partial p}{\partial x} = 0 \right)$  yields good results in the case of heavy and medium load bearings,

but is inappropriate in the case of small load bearings. In this last case, the temperature field is negatively influenced by the errors in the calculation of leaking flows rate out of sides.

For small load bearing, a variant of the differential equation for pressure will have to be used, which should afford the application of boundary conditions specific to small load bearings.

## 6. REFERENCES

- [1]. Hueber, K., Application of Finite Element Methods to Thermohydrodynamic Lubrication, Intl. J. Numerical Methods in Eng., 8, pp 139-165, 1974.
- [2]. Raimondi A., and Boyd, J., A Solution for Finite Journal Bearing and Its Application to Analysis and Design-I, II, III, ASLE Trans., Vol. 1, pp. 159-209, 1958.
- [3]. Khonsari, M. M., Jang, J. Y., and Fillon, M., On the Generalization of Thermohydrodynamic Analyses for Journal Bearings, ASME Journal of Lubrication Technology, Vol. 118, 1996.
- [4]. Ionescu, M., A New Pressure Equation for Finite Length Hydrodynamic Bearings, Acta Tribologica, Volume 5, 1-2, 1997.
- [5]. Constantinescu, V. N., Sliding Bearings, Allerton Press, 1985.
- [6]. Dowson, D., Hudson, J. D., Hunter, B., and March, C. N., An Experimental Investigation of the Thermal Equilibrium of Steadily Loaded Journal Bearings, Proc. Inst. Mech. Engrs., Vol. 131, Part 3B, pp. 70-80, 1966-1967.
- [7]. Ferron, J., Frene, J., and Boncompain, R., A Study of the Thermohydrodynamic Performance of a Plain Journal Bearing Comparison Between Theory and Experiments, ASME Journal of Lubrication Technology, Vol. 105, pp. 422-428, 1983.
- [8]. Mitsui, J., A Study of Thermohydrodynamic Lubrication in a Circular Journal Bearing, Tribology International, Vol. 20, No. 6, pp. 331-341, 1987.