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THEORETICAL AND EXPERIMENTAL ASPECTS CONCERNING THE ANALYTICAL SOLUTION OF HYDRODYNAMIC JOURNAL BEARING LUBRICATION

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ABSTRACT. *The paper presents briefly the main attempts at an analytical solution of the differential equation for pressures proposed by Reynolds [1] and the reasons that have led to their replacement by numerical methods. Subsequently, the differential equation [2] is presented, and its verification in several concrete cases provided by the literature in the field. In its conclusion, the paper presents the advantages and limits of this equation in the analytical estimation of the functional parameters of radial journal bearings.*

KEYWORDS*: analytical solution, journal bearing, film, hydrodynamic, journal bearing*

1. INTRODUCTION

Solving Reynolds' differential equation has been a constant undertaking for all the researchers interested in hydrodynamic lubrication issue. Although the first attempts at solving the equation were analytical, [1, 3] because of the inaccuracy of the solutions or the limited putting into practice thereof, numerical methods were used alongside the computers' increased use. After 1956, the utilization of numerical methods for the solution of the Reynolds equation, Pinkus [4], led to the elimination of analytical approaches. Although the numerical approaches constitute a thorough spectrum of solutions as well as the necessary accuracy for finite length bearings, over the recent years a tendency toward reconsidering the analytic approach has been noted, for resolving both the hydrodynamic and even the thermodynamic issues. Mention must be made here of the papers [5-8], were analytic calculus models are proposed for laminar and turbulent flow of finite bearings.

2. NOMENCLATURE

C = radial clearance (m)
e = eccentricity (m)
h =film thickness (m)
h_0 = film thickness, in which the pressure gradient is zero (m)
L = real length bearing, (m)
L_1 = unitary length, (m), $L_1 = 10^{-5}$ m
p, p_a = pressure, supply pressure at inlet (Pa)
R = shaft radius (m)
T_i =initial temperature, (^{0}C)
T_s = shaft temperature (^{0}C)
V = runner velocity (m/s)
W = load carrying capacity (N)
x, y, z = global coordinates (m)
β = temperature - viscosity coefficient (/^{0}K)
ε = eccentricity ratio
θ = circumferential angular coordinate, (degree)
μ_i, μ =inlet viscosity, lubricant viscosity ($Pa \cdot s$)
ω = angular velocity of the journal (rad/s)

3. THEORETICAL CONSIDERATIONS

In this paper, the film temperature field is considered known, which allows the approach to be restricted to hydrodynamic aspects only. Figures 1 and 2 illustrate the system's coordinates as well as the symbols used in the ensuring relations and reasoning.

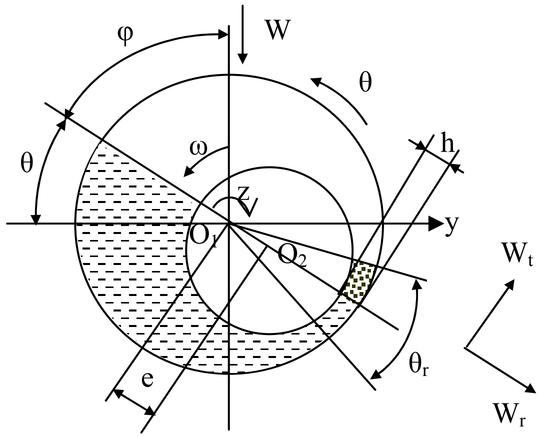

Figure1: *Finite radial bearing*

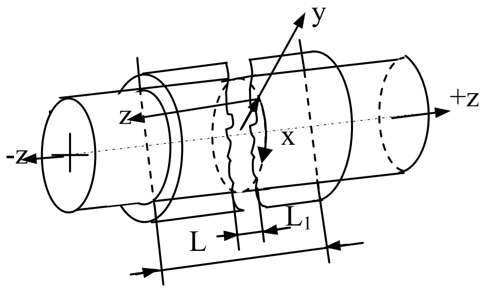

Figure2: *The cross-section of the radial bearing*

The results obtained through numerical solving of Reynolds' equation indicate a fair correspondence with the experimental ones. This proves the unquestionable validity of Reynolds' equation. As for the theory of short bearings [3], the errors of the hydrodynamic parameters are caused by overlooking the first term of the elementary flow with direction x. The analytical treatment of the issue does bring about the overlooking, yet vitiates the solutions to the equation we obtain. In order to eliminate this shortcoming, we need to find and use a function that is able to take on the characteristics of the neglected term. In this respect, in paper [2] was suggested an original argument, which permitted the deduction of the compensating function needed, subsequently a new differential relationship of pressures.

$$\frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\cdot\frac{\partial p}{\partial z}\right)=6\cdot V\cdot\frac{\partial h}{\partial x}\cdot I(\theta) \qquad (1)$$

In the relationship (1), function $I(\theta)$ is the correction function applied to the pressures field:

$$I(\theta)=1-\frac{(L-L_1)\cdot e\cdot\sin\theta}{h_0\cdot R+L\cdot e\cdot\sin\theta} \qquad (2)$$

Equation (1) is integrated twice, the boundary conditions proper to short bearing theory are set and the distribution law for film pressures is obtained:

$$p=p_a-3\cdot\mu\cdot V\cdot\frac{1}{h^3}\cdot\frac{dh}{dx}\cdot\left(\frac{L^2}{4}-z^2\right)\cdot I(\theta) \qquad (3)$$

The calculation of film thickness h_0 is made numerically, assuming the boundary condition:

$$p=p_a,\frac{\partial p}{\partial x}=0 \qquad (4)$$

The quantitative evaluation of the hydrodynamic parameters of the radial bearings may be done analytically, by using relation (3) in the classic corresponding relations, according to papers [2, 7, 8]. The sliding bearing' loads, are calculated with the help of the new pressures equation. The calculation of load W, requires knowledge of the components in direction r and t. Components W_r and W_t are calculated with the following relations:

$$W_r=-R\cdot\int_0^{\pi}(p-p_a)\cdot\cos(\theta)\cdot I(\theta)d\theta \qquad (5)$$

$$W_t=R\cdot\int_0^{\pi}(p-p_a)\cdot\sin(\theta)\cdot I(\theta)d\theta \qquad (6)$$

4. RESULTS AND DISCUSSION

Experimental investigation on thermal effects in journal bearings by Dowson et al., [9], revels that the cyclic variation of the shaft temperature in the circumferential direction is small and that the shaft can be considered as an isothermal component. Thus in relation (3), the medium lubricant viscosity corresponding to the shaft temperature T_s is taken into account.

Consider the Ferron's bearing (cf. Ferron et al., 1983). The bearing specifications are: R=0.05 m, L=0.08 m, C=14.49·10^{-5} m, ω=209.4 rad/sec, β=0.034/^{o}K, W=4000 N, T_i=40^{o}C, μ_i=0.0277 Pas, p_a=70KPa. The bearing under analyses is single-groove, steadily loaded, and the flow is laminar.

By means of the THD method the lubricant film's medium temperature is calculated for all the situations under analysis. Table 1 presents synthetically the experimental data in paper [10] and the theoretical temperatures calculated numerically.

Tabel 1.

Solution / Experimental data, (n=2000rpm)	THD solution	
	ε	T_{shaft} (^{0}C)
1. Ferron, W=2000N	0.438	43.73
2. Ferron, W=2500N	0.473	43.83
3. Ferron, W=3000N	0.529	44.06
4. Ferron, W=4000N	0.577	44.10
5. Ferron, W=5000N	0.654	44.84
6. Ferron, W=6000N	0.677	45.31
7. Ferron, W=7000N	0.708	45.96
8. Ferron, W=8000N	0.721	46.30

According to Dowson et al., [9], the medium temperature of the lubricant film in the convergent-divergent area is considered to be equal to the shaft temperature.

The medium viscosity is then calculated by means of the relation:

$$\mu = \mu_1 \cdot e^{-\beta(T_s - T_i)} \qquad (7)$$

Once the medium viscosity is known, the journals' parameters can be estimated analytically by means of the relation for the distribution of pressures (3).

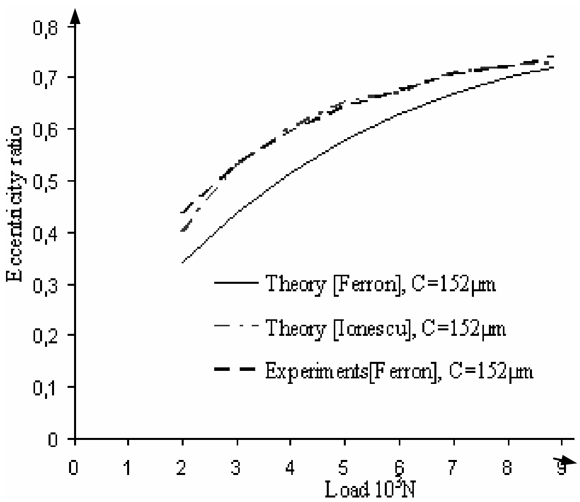

Figure 3: *Eccentricity ratio versus load*

Figure 3 presents the experimental data and, respectively, analytical calculations for loads in a number of concrete situations to be found in paper [10]. The dash-dot line marks the theoretical results obtained analytically by means of equation (3). In all situations presented, the shaft's speed is 2000 rpm.

The load calculation errors are of 10.23% for 2000N, they decrease to 4.38% for 2500N, after which, starting with ε=0.529, which corresponds to a 3000N load, their percentage is not higher than $2 \div 2.5$, compared with the experiment.

In order to improve calculation accuracy for lightly loaded bearings, the following relation has been proposed by Ionescu [11]:

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) = 6V\frac{\partial h}{\partial x}\frac{(L - L_1)\cdot e \cdot \sin\theta}{h_0 \cdot R + L \cdot e \cdot \sin\theta} \qquad (8)$$

Relation (4) affords the application of the lightly loaded bearing condition ($p = p_a, h = h_0$) and can represent the sought-for alternative. Problems arise when the relation is integrated, owing to difficulties concerning the separation of h_0.

5. CONCLUSIONS

The proposed differential equation ensures a good theory-experiment agreement in the case of bearings with heavy and medium loads, but is not satisfactory for those with small loads.

The working times are inconsiderably small by comparison with those required by the numerical methods.

The calculation is easy to perform by comparison with the numerical methods.

For the moment, the equation (8) involves a series of difficulties of a mathematical nature because of the fact that the film thickness h_0 cannot be separated in the calculation.

The equation proposed can be particularized and applied to other types of bearings which work on the same principle of pressure generation.

From a mathematical point of view, the good agreement between theory and experiment offered by the differential equation for pressures proposed in the paper represents a serious argument in favor of the analytical solution of complex differential equations.

6. REFERENCES

[1] Reynolds, O., 1886, "On the Theory of Lubrication and Its Application to Mr. Beauchamp Tower's Experiments Including an Experimental Determination of the Viscosity of Olive Oil", Phi. Trans., 177 (i), pp. 157-234.

[2] Ionescu, M., 1997, "A New Pressure Equation for Finite Length Hydrodynamic Bearings", Acta Tribologica, 5, pp. 7-12

[3] Ocvirk, F. W. and Dubois, G.B., 1952, "Short Bearing Approximation for Full Journal Bearings", NACA TN 2808.

[4] Pinkus, O., 1956, "Analysis of Elliptical Bearings", Transactions ASME, 78, July.

[5] D'Agostino, V., Guida, D., Ruggiero A., Senatore, A., 2001, "An Analytical Study of the Fluid Force in the Finite Journal Bearings: Part I", Lubr. Science-International Journal, pp. 1-10.

[6] D'Agostino, V., della Valle, S., Ruggiero A., Senatore, A., An Approximate Analytical Model for the Analysis of Finite Turbulent Journal Bearings, 3rd Aimeta International Tribology Conferance, 2002.

[7] Ionescu, M., 1999, "Contributions Concerning the Lubrication of the Combined Single Supply Bearings", Ph. D. Thesis, University of Suceava, chapter 2, pp. 71-128.

[8] Ionescu, M., 2004, "Considerations Regarding Thermohydrodynamic Analysis of Journal Bearings", Proceedings of AITC' 04, Rome, Italy, pp. 515-522.

[9] Dowson, D., Hudson, J. D., Hunter, B., and March, C. N., 1966-1967, "An Experimental Investigation of the Thermal Equilibrium of Steadily Loaded Journal Bearings", Proc. Inst. Mech. Engrs., 131, Part 3B, pp. 70-80.

[10] Ferron, J., Frene, J., Boncompain, R., 1983, "A Study of the Thermohydrodynamic Performance of a Plain Journal Bearing Comparison Between Theory and Experiments", ASME Journal of Lubrication Technology, 105, pp. 422-428.

[11] Ionescu, M., 2004, "An, approximation of analysis of laminar flow finite hydrodynamic bearing model", Scientific Bulletin of the University of Timişoara, 1, pag.485-488.