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INERTIA EFFECT IN MICROBEARING GAS FLOW

Nevena Stevanovic[1], Snezana Milicev[1]
[1]Faculty of Mechanical Engineering, University of Belgrade, Belgrade, Serbia

Abstract: *In this paper a two dimensional compressible gas flow in a slider microbearing is investigated analytically. Results are obtained for slip flow conditions which mean that Knudsen number is in the range of 10^{-3}-10^{-1}. Gas flow in microbearing is described by the continuity and Navier-Stokes equations together with the slip boundary conditions at the wall. The gas flow is subsonic and the ratio Ma^2/Re is taken to be of the order of a small parameter. Moreover, it is assumed that the channel cross section varies slowly, which implies that all physical quantities vary slowly in the flow direction. The model solution is treated by developing a perturbation scheme. Pressure and velocity fields are presented. The analytical solution consists of two approximations. The first approximation corresponds to the continuum flow conditions, while the second one involves the influence of inertia as well as rarefaction effect.*

Keywords: *microbearing, slip flow, inertia, analytical solution, low Much number, high Reynolds number.*

1. INTRODUCTION

Gas lubrication is a component of most micro-electro-mechanical systems (MEMS) such as microbearings, micropumps, microvalves or magnetic disk storages [1]. The hard disc industry demands nanometer distances between slider with read/write head and rotating recording disk. The gas slider bearing flow is traditionally modelled by the Reynolds lubrication equation which is derived from the Navier-Stokes and continuity equations under the no slip continuum boundary conditions. The thickness of the lubricating film in microdevices is of the order of the mean free path of gas molecules and the continuum theory is not applicable. A wide range of Knudsen numbers is possible in microdevice flows, but the slip flow regime with $10^{-3} < Kn < 0.1$ is the most frequent. Therefore, solutions for such flow conditions in microbearings are very useful.

Analytical and numerical investigations of the slip gas flow in microbearings were performed. Burgdorfer [2] made the Reynolds equation correction by including the Maxwells first order slip conditions at the wall. Mitsuya [3] set up 1.5-order slip model for ultra thin gas lubrication. Hsia and Domoto [4] developed the second order model by incorporating their second order boundary condition in the Reynolds lubrication equation. They also carried out experiments with different gases in microbearings, and compared the obtained load carrying capacity with analytical results. Sun et al. [5] incorporated expression for the effective viscosity in the Navier-Stokes equation and obtained modified Reynolds equation. Bahukudumbi and Beskok [6] developed semi analytical model for gas lubricated microbearings. They remarked that the viscosity coefficient depends on the Knudsen number. Since the proposed relation is not general, the rarefaction correction parameter is introduced in this relation. Values of the rarefaction correction parameter are defined for certain Knudsen number and surface accommodation parameter values by comparing obtained flow rate results with numerical solutions of the Boltzmann equation under the same conditions [7], [8]. Finally, the derived function of the viscosity coefficient is introduced in the model, and the new modified Reynolds equation is obtained. Liu and Ng [9] analysed the posture effects of a slider air bearing and the influence of the lower plate velocity on the pressure distribution and velocity field with a direct simulation Monte Carlo method.

The model developed in this paper for microbearing gas flow is based on already verified results for a pressure driven gas flow in a microchannel with slowly varying cross section [10] and low Reynolds number gas flow in the microbearing [11]. The low Much number gas flow is considered, which enables a definition of the small parameter $\varepsilon = \gamma Ma^2 / Re$. Moreover, it is assumed that the channel cross section varies slowly, which also implies that all physical quantities vary slowly in the flow direction. All these assumptions together with the defined relations between the Reynolds, Much and Knudsen number and the small parameter ε, enable a precise estimation of each term in the non-dimensional governing equations, as well as in the boundary conditions. In the solving procedure, the pressure and velocity are expressed as the perturbation series of the Knudsen number. The system of nonlinear second order differential equations is obtained, and it is solved numerically.

2. MODEL STATEMENT

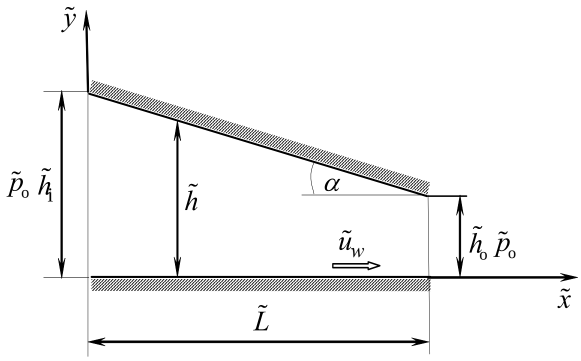


Figure 1. Slider microbearing geometry

An isothermal two-dimensional and compressible gas flow in microbearing (as presented in figure 1) is considered and describer with a continuity equation, Navier-Stokes equation for compressible gas flow, equation of state and a slip boundary condition at the wall. These equations are transformed into a non-dimensional form by the introduction of the following scales: exit microbearing height $\tilde{h}_o$ for all lengths, wall velocity $\tilde{u}_w$ for all velocity components, and the pressure and density are scaled with the corresponding values $\tilde{p}_o$ and $\tilde{\rho}_o$ at the channel outlet cross section. Then the assumption of the low Mach number flow conditions enables a definition of the small parameter

$$\varepsilon = \gamma M_o{}^2 / Re_o \tag{1}$$

where $\gamma = c_p / c_v$ is the ratio of specific heats, M_o is the referent Mach number value defined as

$$M_o = \tilde{u}_w \Big/ \sqrt{\gamma \, \tilde{p}_o / \tilde{\rho}_o} \tag{2}$$

and Re_o is the referent Reynolds number

$$Re_o = \tilde{\rho}_o \tilde{u}_w \tilde{h}_o / \tilde{\mu} \tag{3}$$

Dynamic viscosity $\tilde{\mu}$ is assumed to be constant. Now, the expression for the small parameter ε follows: $\varepsilon = \tilde{\mu}\tilde{u}_w / \left(\tilde{p}_o \tilde{h}_o \right)$.

The assumption of the slowly varying channel cross section $\alpha \approx \varepsilon << 1$, where α is the channel wall inclination (figure 1) implies that all flow parameters change vary slowly in the x- axes direction, which is explicitly expressed by the introduction of the slow coordinate $\xi = \varepsilon x$. Also, the crosswise velocity component v is much smaller than the streamwise component u, which leads to the following relation: $v(x, y) = \varepsilon V(\xi, y)$, $V = O(1)$.

The continuity equation, the Navier-Stokes equations for the stream-wise and cross-wise directions and the equation of state in non-dimensional form are:

$$\partial(pu)/\partial\xi + \partial(pV)/\partial y = 0 \tag{4}$$

$$\gamma M_o^2 p \left(u \frac{\partial u}{\partial \xi} + V \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial \xi} + \frac{\partial^2 u}{\partial y^2} + O\left(\varepsilon^2\right) \tag{5}$$

$$\frac{\partial p}{\partial y} = O\left(\varepsilon^2\right) \tag{6}$$

$$p = \rho \tag{7}$$

Further, all nondimensional parameters are denoted without a bar, i.e. pressure as p, stream-wise velocity component as u, etc.

Using the same scale for velocity and length, the Maxwell's first order boundary condition are expressed for lower mowing and upper fixed wall in non-dimensional form:

$$y = 0: u\text{-}u_w = \frac{2 - \sigma_{v1}}{\sigma_{v1}} \left[\frac{Kn_o}{p} \frac{\partial u}{\partial y} + O\left(Kn_o^2\right) \right], V = 0 \tag{8}$$

$$y = h(\xi): u = \frac{2 - \sigma_{v2}}{\sigma_{v2}} \left[-\frac{Kn_o}{p} \frac{\partial u}{\partial y} + O\left(Kn_o^2\right) \right], V = u \frac{\mathrm{d}h}{\mathrm{d}\xi} \tag{9}$$

where σ_{v1} and σ_{v2} are the tangential momentum accommodation coefficient for lower and upper walls. The local Knudsen number defined as $Kn = \tilde{\lambda} / \tilde{h}_o$ is expressed in the boundary condition through the referent Knudsen number Kn_o

$$Kn_o = \tilde{\lambda}_o / \tilde{h}_o = \sqrt{\frac{\pi\gamma}{2}} \frac{M_o}{Re_o} \tag{10}$$

For isothermal flow conditions, the local value of the mean free path of molecules $\tilde{\lambda}$ is inversely proportional to pressure which means that $\tilde{\lambda} = \tilde{\lambda}_o / p$, where $\tilde{\lambda}_o$ is molecules mean free path referent value at exit cross section. This leads to the relation $Kn = Kn_o / p$ among the local and referent Knudsen number which enables to express the boundary conditions as a function of the referent Knudsen number.

The presumption of extremely subsonic flow in the slip regime enables the relation between the Mach and Knudsen numbers and the small parameter ε: $\gamma M_o^2 = \beta\varepsilon^m$, $\beta = O(1)$ and $Kn_o = \eta\varepsilon^n$, $\eta = O(1)$. Due to the relation between the Reynolds, Much and Knudsen number (10) and the definition of the small parameter ε, the exact expression for the Reynolds number and relations among introduced parameters m and n, as well as β and η follows: $Re_o = \beta\varepsilon^{m-1}$, $2n + m = 2$ and $\eta = \sqrt{\pi/2\beta}$. Supposition of the low Mach and Knudsen number flow, limited m and n values to positive domain, which together with the relation $2n + m = 2$ gives that this parameters must be in the following ranges: $0<m<2$ and $0<n<1$. In this frame two characteristic problems could be analysed: $Re < 1$ if $1<m<2 \Rightarrow 0<n<1/2$ and $Re > 1$ if $0<m<1 \Rightarrow 1/2<n<1$. In derivation of the Reynolds equation for lubrication theory, the inertia term is neglected which corresponds to the low Reynolds number case which was already obtained [11]. In this paper solution for $Re > 1$ is presented. Values for parameters m and n are chosen to enable attendance of the inertia effect together with the rarefaction: $m=n=2/3$. The relations for the non-dimensional numbers are: $Re_o = \beta\varepsilon^{-1/3}$, $\gamma M_o^2 = \beta\varepsilon^{2/3}$, $Kn_o = \eta\varepsilon^{2/3}$.

All dependant variables from equations (4)-(7), i.e. pressure and velocity components, are presented in the form of perturbation series

$$F = F_0 + \varepsilon^{2/3} F_{2/3} \tag{11}$$

where F_0 is the solution for the flow with no-slip boundary conditions, and $F_{2/3}$ comprise the corrections for the inertia effect and the slip on the wall. The systems of equations for two approximations together with corresponding boundary conditions are obtained by substitution perturbation series for pressure and velocities in equations (4)-(7) and (8)-(9). In order to catch up the slip effect already in the second approximation, the power for small parameter in the second term on the r.h.s. of equation (11) is the same as for the Knudsen number ($\varepsilon^{2/3}$). As inertia in equation (5) is of the order $\gamma M_o^2 = \beta\varepsilon^{2/3}$, for the perturbation series in equation (11) the inertia effect is included also in the second approximation. The velocity and pressure perturbation expressions, in the form of equation (11), are introduced in the continuity equation (4), the momentum conservation equation (5) and the boundary conditions equations (8) and (9). From these equations, the terms of the order $O(1)$ and $O(\varepsilon^{2/3})$ are extracted, and the following sets of equations are obtained

- for $O(1)$

$$\frac{\partial p_0}{\partial \xi} = \frac{\partial^2 u_0}{\partial y^2} \tag{12a}$$

$$\frac{\partial (p_0 u_0)}{\partial \xi} + \frac{\partial (p_0 V_0)}{\partial y} = 0 \tag{12b}$$

$$y = 0: \quad u_0 = 1, \quad V_0 = 0 \tag{12c}$$

$$y = h(\xi): \quad u_0 = 0, \quad V_0 = 0 \tag{12d}$$

- for $O(\varepsilon^{2/3})$

$$\beta p_0 \left[u_0 \frac{\partial u_0}{\partial \xi} + V_0 \frac{\partial u_0}{\partial y} \right] = -\frac{\partial p_{2/3}}{\partial \xi} + \frac{\partial^2 u_{2/3}}{\partial y^2} \tag{13a}$$

$$\frac{\partial (p_0 u_{2/3} + p_{2/3} u_0)}{\partial \xi} + \frac{\partial (p_0 V_{2/3} + p_{2/3} V_0)}{\partial y} = 0 \tag{13b}$$

$$y = 0: u_{2/3} = \frac{2 - \sigma_{v1}}{\sigma_{v1}} \frac{\eta}{p_0} \frac{\partial u_0}{\partial y}, V_{2/3} = u_{2/3} \frac{\mathrm{d}h}{\mathrm{d}\xi} = 0 \tag{13c}$$

$$y = h(\xi): u_{2/3} = -\frac{2 - \sigma_{v2}}{\sigma_{v2}} \frac{\eta}{p_0} \frac{\partial u_0}{\partial y}, V_{2/3} = u_{2/3} \frac{\mathrm{d}h}{\mathrm{d}\xi} \tag{13d}$$

The solution procedure for each system of these equations is the same. The approximations of the stream-wise velocity component u_0, $u_{2/3}$ are derived from the corresponding momentum equations (12a) and (13a). The pressure approximations p_0, $p_{2/3}$ are derived from the corresponding continuity equations (12b), (13b).

Velocity and pressure equations for the first four approximations are obtained in the following form

- the first approximation

$$u_0 = 1 - \frac{y}{h}\left(1 + p_0' \frac{h^2}{2\xi_L}\right) + p_0' \frac{y^2}{2\xi_L} \tag{14}$$

$$\left[h^3\left(p_0 p_0'\right)\right]' - 6\xi_L\left(p_0 h\right)' = 0 \qquad (15)$$

- the second approximation

$$u_{2/3} = \left[\frac{2-\sigma_{v1}}{\sigma_{v1}}\left(\frac{1}{\delta^2} - \frac{1}{y\delta} + \frac{p_0'}{2\xi_L} - \frac{\delta p_0'}{2y\xi_L}\right)\right]\frac{y\eta}{p_0}$$

$$+\frac{2-\sigma_{v2}}{\sigma_{v2}}\left(\frac{1}{\delta^2} - \frac{p_0'}{2\xi_L}\right)\frac{y\eta}{p_0} + \frac{p_{2/3}'\delta^2}{2\xi_L}\left(\frac{y^2}{\delta^2} - \frac{y}{\delta}\right) +$$

$$+\beta\left\{A\frac{\delta^3}{6}\left(\frac{y^3}{\delta^3} - \frac{y}{\delta}\right) + B\frac{\delta^4}{24}\left(\frac{y^4}{\delta^4} - \frac{y}{\delta}\right) -\right.$$

$$\left.-C\frac{\delta^5}{60}\left(\frac{y^5}{\delta^5} - \frac{y}{\delta}\right) + E\frac{\delta^6}{360}\left(\frac{y^6}{\delta^6} - \frac{y}{\delta}\right)\right\} \qquad (16)$$

where:

$$A = \frac{\left(p_0\delta\right)'}{\xi_L\delta^2} - \frac{p_0^2}{2\xi_L^2}\left(\frac{p_0'\delta}{p_0}\right)'$$

$$B = \frac{p_0^2 p_0'\delta}{4\xi_L^3}\left(\frac{p_0'\delta}{p_0}\right)' - \frac{\left(p_0\delta\right)'}{\xi_L\delta^3} + \frac{3p_0^3}{2\xi_L^2}\left(\frac{p_0'}{p_0^2}\right)'$$

$$C = \frac{p_0^3}{\xi_L^2}\left(\frac{p_0'}{p_0^2}\right)'\left(\frac{p_0'\delta}{2\xi_L} + \frac{1}{\delta}\right), \; E = \frac{p_0^3 p_0'}{\xi_L^3}\left(\frac{p_0'}{p_0^2}\right)'.$$

$$\left[\frac{\delta^3}{12}\left(p_0 p_{2/3}\right)'\right]' = \xi_L\frac{\left(p_{2/3}\delta\right)'}{2} - \frac{2-\sigma_{v2}}{\sigma_{v2}}\eta\left(\frac{p_0'\delta^2}{2}\right)' -$$

$$-\left[\beta\xi_L p_0\left(\frac{A\delta^4}{24} + \frac{B\delta^5}{80} - \frac{C\delta^6}{180} + \frac{E\delta^7}{1008}\right)\right]' \qquad (17)$$

where the stream-wise coordinate is $X = \xi/\xi_L$. The prime denotes $\mathrm{d}/\mathrm{d}X$, while h is the channel cross section in dependence on X defined as: $h(X) = h_\mathrm{i} - X(h_\mathrm{i} - 1)$, where h_i is nondimensional parameter defined as ratio of the inlet and outlet microbearing height $h_\mathrm{i} = \tilde{h}_\mathrm{i}/\tilde{h}_\mathrm{o}$.

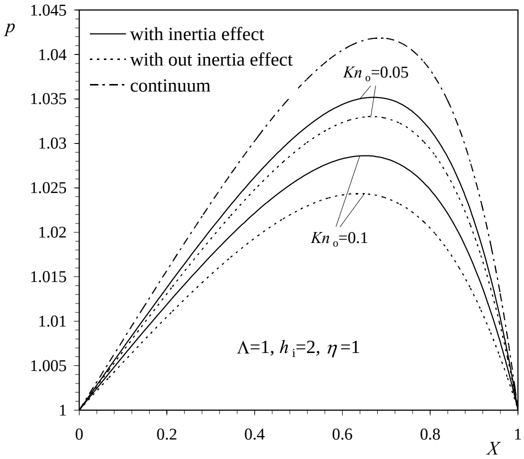
The channel length expressed by the slow coordinate is: $\xi_L = \varepsilon\,\tilde{L}/\tilde{h}_\mathrm{o} = \tilde{\mu}\tilde{u}_\mathrm{w}\tilde{L}/\left(\tilde{p}_\mathrm{o}\tilde{h}_\mathrm{o}^2\right)$. The bearing number definition is $\Lambda = 6\tilde{\mu}\tilde{u}_\mathrm{w}\tilde{L}/\left(\tilde{p}_\mathrm{o}\tilde{h}_\mathrm{o}^2\right)$ and it is evident relation with parameter ξ_L

$$\xi_L = \Lambda/6 \qquad (18)$$

This means that pressure distribution and velocity field which are obtained from equations (14-17) is defined by bearing number Λ, referent Knudsen number Kn_o, channel geometry h and parameter η. The system of the two second order differential equations (15) and (17) that enables the prediction of pressure along the microbearing, demands four boundary conditions at the channel inlet - p_0, $p_{2/3}$, p_0', $p_{2/3}'$. But, the first derivate of pressure is not known. This problem is overcome by using the known pressure at channel outlet instead of the first pressure derivate at the inlet, which imposed the application of the shooting method for the solving of system of equations. The boundary conditions for pressure, prescribed at the inlet and outlet are $X = 0,\; p = p_0 = 1$, $p_{2/3} = 0$ and $X = 1,\; p = p_0 = 1$, $p_{2/3} = 0$.

3. RESULTS AND DISCUSSION

In figure 2 the pressure distributions in the microbearing witch are obtained with inertia effect contribution and with no inertia effect $\beta = 0$ are presented. The results are obtained for the ratio of inlet to outlet heights $\tilde{h}_\mathrm{i}/\tilde{h}_\mathrm{o} = 2$, two bearing number values, $\Lambda = 1$ and $\Lambda = 10$, two Knudsen number values $Kn_\mathrm{o} = 0.1$ and $Kn_\mathrm{o} = 0.05$. Besides the bearing number Λ, the inlet to outlet ratio $\tilde{h}_\mathrm{i}/\tilde{h}_\mathrm{o}$ and the Knudsen number, which are usually defined flow conditions in the microbearing according with the Reynolds lubrication theory which negligible the inertia influence, in this model the parameter η is also need for the solving of the system of differential equations (14), (15), (16), (17). This is the consequence of the incorporation of the inertia effect in the model. All results presented in this paper are obtained for $\eta = 1$. It is evident that the inertia leads to the pressure increase in the microbearing. On figure 2 are also presented results obtained for continuum flow condition ($Kn = 0$) and it is obvious that slip effect at the wall leads to the lower pressure in the microbearing. Figure 2 shows excellent agreement of presented model with no inertia effect with Fukui and Kaneko numerical solution of the Boltzmann equation [7], [8], obtained also with inertia omit. The pressure distribution along microbearing enables the calculation of velocity field from equations (14) and (16). In figures 3 and 4 velocity profiles for the two flow conditions defined with $Kn_\mathrm{o} = 0.1$, $\tilde{h}_\mathrm{i}/\tilde{h}_\mathrm{o} = 2$, $\Lambda = 1$, $\eta = 1$ and $Kn_\mathrm{o} = 0.1$, $\tilde{h}_\mathrm{i}/\tilde{h}_\mathrm{o} = 2$, $\Lambda = 10$, $\eta = 1$ are presented.



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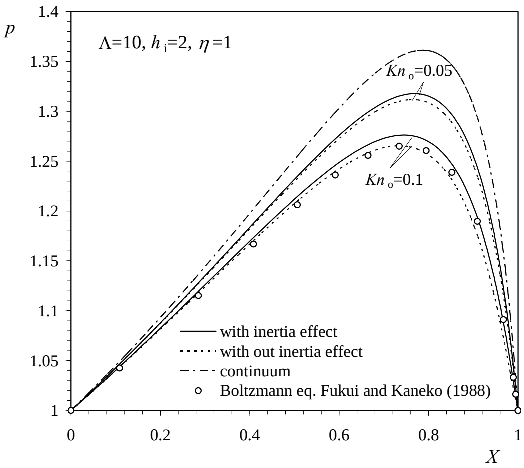


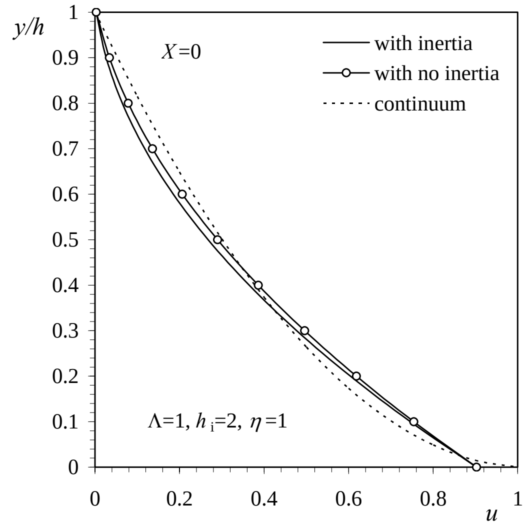
b)

Figure 2. Pressure distribution in microbearing for $Kn_o = 0.1$, $\tilde{h}_i / \tilde{h}_o = 2$, and $\eta = 1$ different bearing number: a) $\Lambda = 1$, b) $\Lambda = 100$

The full line presents velocity profiles with inertia influence, while the full lines with circles present velocity profile with no inertia effect. For each flow conditions the velocity profile at the inlet, outlet and about $X = 0.8$ is obtained. The dashed lines in figures 3 and 4 present velocity profiles for the continuum conditions. The difference between velocity profiles for the continuum and slip flow conditions is evident, as well as between velocity profiles for slip at the wall obtained with inertia effect and obtained with neglecting inertia. The inertia influence is more pronounced for the lower bearing number value.

4. CONCLUSION

The new analytical method, already verified for the pressure driven gas flow in microchannels and lubrication in the microbearing for the $Re < 1$ flow conditions. The small parameter is defined by equation (1), and all nondimensional numbers (the Reynolds, Much and Knudsen number) are expressed with it. In this way, the exact relation between all these numbers is introduced in the continuity and momentum equations, together with



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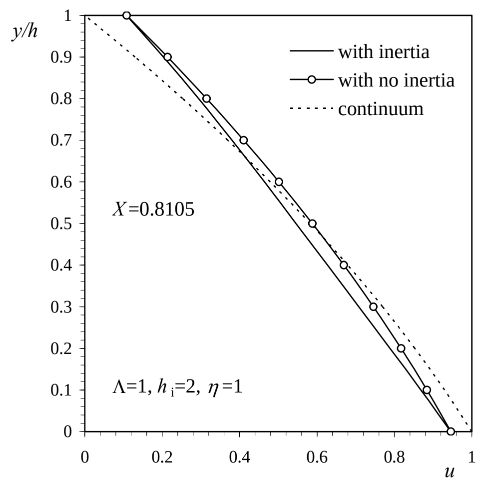

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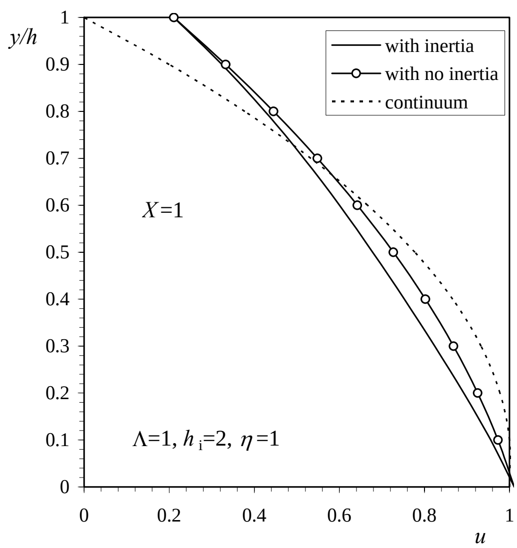
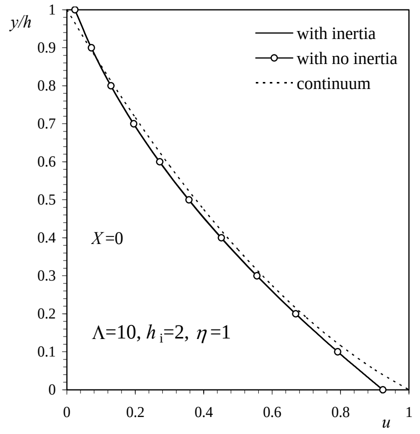


c)

Figure 3. Velocity distribution in microbearing for $Kn_o = 0.1$, $\tilde{h}_i / \tilde{h}_o = 2$, $\Lambda = 1$ and $\eta = 1$ at three streamwise positions: a) *X*=0, b) *X*=0.8105, c) *X*=1.



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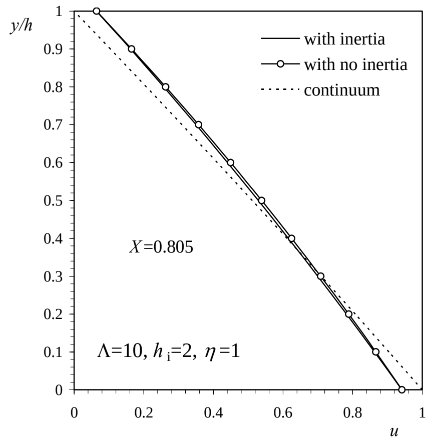

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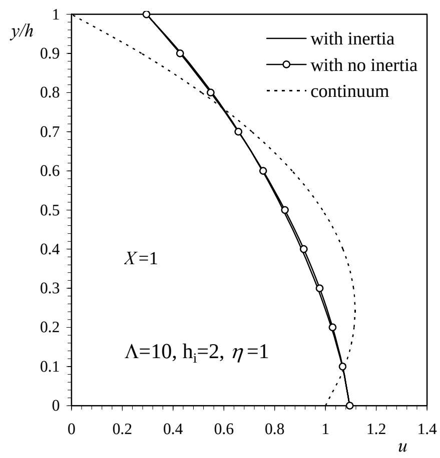

c)

Figure 4. Velocity distribution in microbearing for $Kn_o = 0.1$, $\tilde{h}_i/\tilde{h}_o = 2$, $\Lambda = 10$ and $\eta = 1$ at three streamwise positions: a) *X*=0, b) *X*=0.805, c) *X*=1.

the boundary relations, and the precise estimate of each term contribution is made under presumed conditions. This is the important conception that is incorporated in the model, and it significantly improves method reliability. Solutions of velocity and pressure are supposed by perturbation series in the Knudsen number and the results with two approximations are obtained. Obtained analytical solution is in the form of system of the second order nonlinear differential equations that is solved numerically. For determined flow conditions defined by the referent Knudsen number Kn_o, the bearing number Λ, the channel geometry $\tilde{h}_i/\tilde{h}_o$ and parameter η, the pressure distribution and the velocity field are obtained. This new analytical method applied to the $Re > 1$ flow conditions in the microbearing enable obtaining results which include inertia effect. By comparing it with results with out inertia, for the same flow conditions, influence of inertia is obvious. This method enables extension work towards nonisothermal microbearing gas flow.

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