



## EVALUATION OF MASS-CONSERVATIVE MODELS FOR THE TRIBOLOGICAL ANALYSIS OF POROUS BEARINGS

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**Abstract:** Two theoretical mass-conservative models for the tribological evaluation of porous journal bearings are investigated rigorously. The first model supposes that the lubricating fluid film is fully contiguous in the range between two angles, i.e. effects due to cavitation are excluded. The fluid pressure distribution is determined numerically by solving a modified Reynolds equation which governs the fluid film and accounts for the porous surface and the well-known Darcy's law that describes the flow through the porous matrix in the usual manner. These equations are supplemented with the integral angular momentum equation applied to the fluid film and the integral mass balance between the flow rates of the lubricant entering into and leaking from the clearance, respectively. Considering the second theoretical model, we again solve the Laplace equation governing the pressure distribution due to Darcy's law that holds in the porous bearing seat, but adopt a modification of the Elrod's model in order to study the flow in the clearance.

**Keywords:** hydrodynamic lubrication, journal porous bearings, cavitation, angular momentum equation, Elrod's model

### 1. INTRODUCTION

Self lubricated porous journal bearings are used in a lot of applications due to several advantages: they are cheaper than rolling bearings and produce less noise. Therefore they can be used in a lot of applications, like household appliances, all sorts of fans, and tool kits. Numerous articles in the literature present experimental tests in order to evaluate the cavitation zone in the fluid film [1] or the film thickness and friction torque of these bearings [2]. Theoretical models are also presented based on different assumptions:

- the lubricating fluid film is fully contiguous in the range between two angles and the integral angular momentum equation is applied to the fluid film [3];

- the cavitation model of Elrod is applied in the clearance region [4], [5]

Recently the flexibility of porous liner was introduced in the numerical computations [7] [8].

Due to the fact that the model proposed by Kaneko [3] was not compared to any experimental data, a more rigorous analysis of the above models is needed.

The present paper compares two mass conservative models [3], [5] with experimental data [2] in order to establish which of the two models is more appropriate for the modelling of the tribological performance of self lubricated porous bearings and more suited for further developments.

### 2. GOVERNING EQUATIONS

#### 2.1 Kaneko's model

The first model supposes that the lubricating fluid film is fully contiguous in the range  $\theta_1 < \theta < \theta_2$  (Figure 1), i.e. effects due to cavitation are excluded [3]. The fluid pressure distribution is determined numerically by solving a modified Reynolds equation which governs the fluid film and

accounts for the porous surface and Darcy's law that describes the flow through the porous matrix in the usual manner.

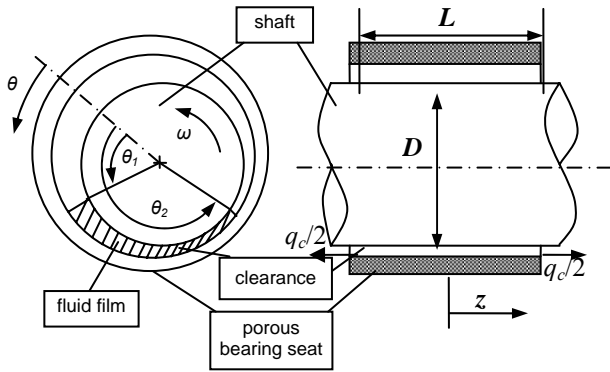


Figure 1. Schematic of the bearing

### Modified Reynolds equation in the fluid film

The Reynolds equation in the fluid film was modified in order to account for the permeability of the bearing housing:

$$\frac{\partial}{\partial \theta} \left( \bar{h}^3 \frac{\partial \bar{p}}{\partial \theta} \right) + \left( \frac{D}{L} \right)^2 \frac{\partial}{\partial z} \left( \bar{h}^3 \frac{\partial \bar{p}}{\partial z} \right) = \frac{\partial \bar{h}}{\partial \theta} - 12\phi \frac{\partial \bar{p}}{\partial r} \Big|_{r=1} \quad (1)$$

### Equation in porous media

The equation describing the lubricant flow in the porous media is the Darcy's equation which in terms of pressure is the Laplace equation:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \bar{p}_p}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \bar{p}_p}{\partial \theta^2} + \left( \frac{D}{L} \right)^2 \frac{\partial^2 \bar{p}_p}{\partial z^2} = 0 \quad (2)$$

### Continuity of flow

The flow that enters in the film from the porous matrix is equal to the axial leakage flow through the clearance gap.

$$q_{in} = q_c \quad (3)$$

### Momentum equation

Applying the momentum of momentum equation yields (the torque radius being constant and the control volume is the fluid film):

$$M_{\theta 1} - M_{\theta 2} - M_{\theta c} = 0 \quad (4)$$

where:

$$M_{\theta 1} = 2 \int_0^{L/2} \int_0^{h_{\theta 1}} \rho (u_{\theta|_{\theta=\theta 1}})^2 dy dz \quad (5)$$

$$M_{\theta 2} = 2 \int_0^{L/2} \int_0^{h_{\theta 2}} \rho (u_{\theta|_{\theta=\theta 2}})^2 dy dz \quad (6)$$

$$M_{\theta c} = 2r_i \int_{\theta 1 0}^{\theta 2 h} \rho (u_{\theta} \cdot u_z) \Big|_{z=L/2} dy d\theta \quad (7)$$

### Model by Giudicelli and Elrod

#### Model describing the fluid film

The Elrod's model was employed in order to solve the Reynolds equation in the fluid film. In the zone with negative pressures, a mixture of oil, oil vapour and air is built.

The universal differential equation in the fluid film contains the oil fraction  $\sigma$  as variable. The relation between  $p$  and  $\sigma$  is:

$$p = p_c + g\beta(\sigma - 1) \quad (8)$$

where:

$$g = 1 \text{ and } \sigma = 1 \text{ when } p > p_c$$

$$g = 0 \text{ and } \sigma \leq 1 \text{ when } p = p_c$$

Then the modified Reynolds equation is:

$$g \nabla_{\theta, z} \cdot (h^3 + 6Kh) \nabla_{\theta, z} p = 6\eta\omega \left[ g \frac{dh}{d\theta} + (1-g) \frac{\partial(\sigma h)}{\partial \theta} \right] - 12K \frac{\partial \bar{p}}{\partial r} (r_i, \theta, z) \quad (9)$$

### Equation in porous media

The equation in the porous media is again the Darcy's equation as in the previous model.

The coupling between the two equations above was realized by considering the continuity of mass flow through the interface between the fluid film and the porous media.

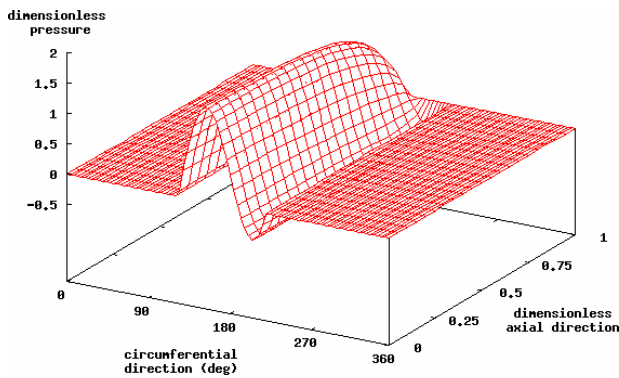
## 3. METHOD OF SOLUTION

The system of differential equations was solved by using the finite-difference method, where second-order accuracy was achieved. The resulting system of algebraic equations was solved iteratively by adopting the Gauss-Seidel method.

## 4. RESULTS

The pressure distribution was computed using the two models presented above for the following input parameters:  $\varepsilon = 0.7$ ,  $D = 19\text{mm}$ ,  $L = 33\text{mm}$ ,  $c = 32\mu\text{m}$ ,  $D_b = 25\text{mm}$ ,  $\phi = 15 \times 10^{-14}\text{m}^2$ ,  $\eta = 0.035 \text{ Pa s}$ ,  $n = 870 \text{ r.p.m.}$ ,  $p_c = -0.1\text{bar}$ .

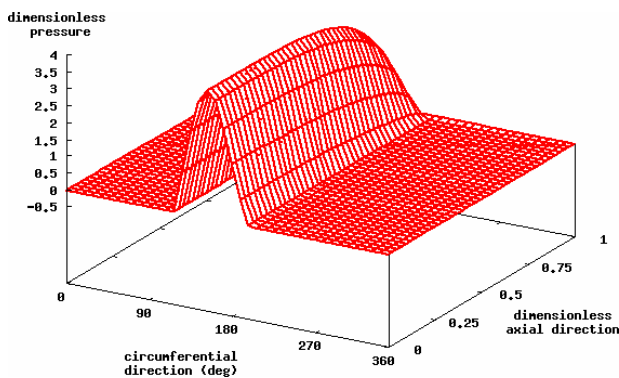
The pressure distribution using the model proposed by Kaneko is presented in Figure 2.



**Figure 2.** Pressure distribution in the fluid film – Kaneko’s model

One clearly observes a small region in the divergent zone where negative pressures appear.

The pressure distribution obtained by using the Elrod’s model is presented in Figure 3.



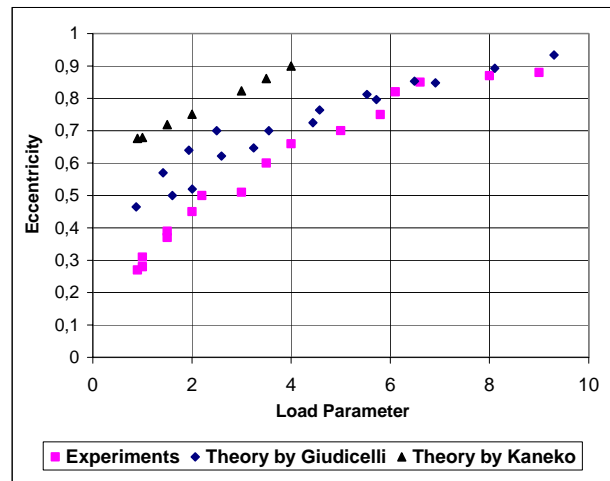
**Figure 3.** Pressure distribution in the fluid film – Elrod’s model

It can be seen that the maximal pressure is double in the case of the Elrod’s model. This could be due to the fact that the Kaneko’s model allows negative pressures in the computation procedure.

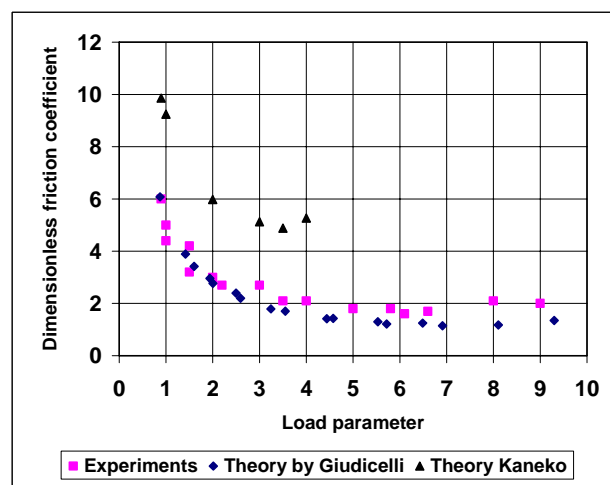
The variation of the eccentricity as a function of the load parameter is presented in Figure 4, where the two theoretical models are compared with the experimental data. All the experimental data are clearly provided and discussed vividly in the literature cited, except for the values of temperatures and, consequently, the viscosities in the fluid film. Hence, rather rough estimates for the latter had to be made.

In passing we note that in case of the Kaneko’s model, numerical difficulties were encountered due to the relatively high values of the eccentricities obtained. Using the Kaneko’s model higher eccentricities are obtained compared to the experimental data. In case of the Elrod’s model the obtained eccentricities are slightly higher, but the results better approximate the experimental data.

The variation of the friction coefficient as a function of load parameter is presented in Figure 5.



**Figure 4.** Eccentricity of the shaft versus load parameter



**Figure 5.** Friction behaviour versus load parameter

We can remark that the friction coefficients obtained by employing the Kaneko’s model are much higher than the corresponding experimental data. In the case of the Elrod’s model the friction coefficients are slightly higher but the results are qualitatively quite similar.

## 5. CONCLUSION

Two theoretical models concerning the evaluation of frictional properties of unsupplied porous bearings have been evaluated in this paper.

It is found that the results based on the cavitation algorithm proposed by Giudicelli and Elrod agree considerably better with the experimental findings.

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## REFERENCES

- [1] A. L. Braun, *Porous Bearings*, Tribology International, Vol. 15, No.5, pp. 235-242, 1982
- [2] M. O. A. Mokhtar, M. Rafaat and G.S.A. Shawki, 1984, *Experimental Investigations Into the Performance of Porous Journal Bearings*, S.A.E. Paper No. 840097, 1984
- [3] S. Kaneko, H. Takabatake, K. Ito, *Numerical Analysis of Static Characteristics at Start of Operation in Porous Journal Bearings with Sealed Ends*, Journal of Tribology, Vol. 121, pp.62-68, 1999
- [4] Elrod, H.G., *A cavitation algorithm*, ASME Journal of Lubrication Technology, Vol. 103, No.2, pp.350-354, 1981
- [5] Giudicelli, B., *Modeles hydrodynamiques conservatifs pour des paliers autolubrifiants poreux en regime permanent*, These de Doctorat, Institut National des Sciences Appliquees de Lyon, France, 1993
- [6] M. Meurisse, B. Giudicelli, *A 3D Conservative Model for Self-Lubricated Porous Journal Bearings in a Hydrodynamic Steady State*, Journal of Tribology, Vol. 121, pp. 529-537, 1999
- [7] A. A. Elsharkawy, L. H. Guedouar, *Hydrodynamic lubrication of porous journal bearings using a modified Brinkman-extended Darcy model*, Tribology International, Vol. 34, pp.767-777, 2001
- [8] A. Marinkovic, F. Franek, A. Pauschitz, *Simulation and optimum design of porous journal metal bearing under elasto-hydrodynamic lubrication*, World Tribology Conference, Washington, 2005

## NOTATIONS

$c$  – bearing clearance

$D$  – shaft diameter

$D_b$  – bearing outer diameter

$\bar{F}$  – load parameter,  $\bar{F} = \frac{Fc^2}{\eta U r_i^2 L}$

$g$  – switch function

$h$  – film thickness

$L$  – bearing seat width

$M_{\theta_1}$  – circumferential momentum flow rate across oil-film surface at inlet end of oil region ( $\theta = \theta_1$ )

$M_{\theta_2}$  – circumferential momentum flow rate across oil-film surface at trailing end of oil region ( $\theta = \theta_2$ )

$M_{\theta_c}$  – circumferential momentum flow rate across oil-film surface at both axial ends

$p$  – fluid pressure

$p_c$  – cavitation pressure

$q_{in}$  – flow from porous matrix in the fluid film

$q_c$  – axial leakage flow from both ends through the clearance gap

$r$  – radial coordinate

$r_i$  – internal radius of bearing seat

$z$  – axial coordinate

$\bar{h}$  – dimensionless film thickness,  $h/c$

$\bar{p}$  – dimensionless pressure,  $\bar{p} = \frac{c^2 p}{r_i^2 \eta \omega}$

$\bar{r}$  – dimensionless radial coordinate,  $r/r_i$

$\bar{z}$  – dimensionless axial coordinate,  $z/(L/2)$

$\varepsilon$  – bearing eccentricity

$\eta$  – oil viscosity

$\phi$  – material permeability

$\theta$  – angular coordinate

$\sigma$  – oil film fraction