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EULER'S LAW IN THE ASPECT OF THE GENERAL LAW OF CONTACT INTERACTION OF TRIBOLOGY

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Abstract: *The paper studiesEuler's law of friction between flexible thread and immovable cylindrical surface on the base of the contact interaction in tribology. Method and experimental procedure have been developed related to this study, so two improved forms of Euler's law taking into account the duration of the static contact between the flexible element and the fixed surface have been established.*

Keywords: *Tribomechanics, Law of contact interaction, friction force, Euler's law*

1. INTRODUCTION

The general law of contact interaction is a quantitative expression of the contact approach and the model of the functional atom related to all events in the behavior of contact systems. Roughly speaking, this law is the hearth of the interdisciplinary paradigm of tribology. Above facts are exemplified with the conduct of contact systems in different fields of nature, techniques and society [1]÷[5].

The paper aims to analyze and improve Euler's formula on friction between flexible element (a thread or wire) and immovable cylindrical or drum surface [6], [7] by the use of the potentialities of the general law of contact interaction.

This case is worth to be submitted to individual discussion, having in view that Euler's law is recognized as perfect success of mechanics in the field of friction between bodies [6], [7]. Moreover, Euler's law faultlessness is also adopted by the researchers in tribomechanics [8].

2. ANALYSIS

Let the flexible element (metal wire, polymer or textile thread) is put over immovable cylinder with hung equal loads P on both ends, then the initial (starting) friction force T according to Euler's law is calculated by the formula:

$$F = P.e^{\mu.\varphi} = P + T \qquad (1)$$

i.e.

$$T = P\left(e^{\mu.\varphi} - 1\right) \qquad (2)$$

where μ is the initial coefficient of friction, and φ - the enfold angle between thread and cylinder (Figure 1).

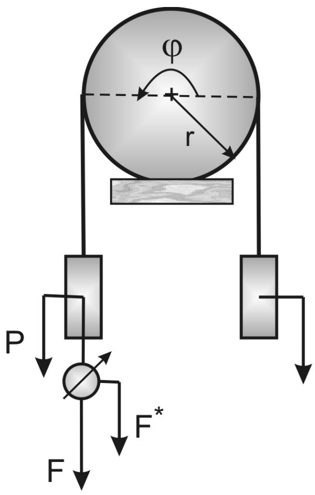

Figure 1. Scheme for the study of the initial friction force

According to equation (2) the friction force depends only on the material of the thread by means of μ, on the load P and the angle φ.

The initial friction force T as a reaction is in general function of the load P and the angle φ for each concrete thread for one and the same cylinder. The load P in that given case is force perturbation, and φ - geometrical perturbation. What is characteristic for Euler's law is that the force T does not depend on the radius r of the cylinder, on the direction of slipping, on the temperature and humidity of the environment, on the duration of the contact, etc.

2.1 Theoretical

Let look at the problem from the viewpoint of the general law of contact interaction in the form:

$$\frac{dR}{R}=\eta_1\frac{dA_1}{A_1}+\eta_2\frac{dA_2}{A_2}+...+\eta_n\frac{dA_n}{A_n}=\sum_i^n\eta_i\frac{dA_i}{A_i} \quad (3)$$

Then the total reaction R is superimposed by the reactions caused by the perturbations A_i with communication potentials η_i.

The general law (3) for the case of Euler (2) has the form:

$$\frac{dT}{T}=\eta_1\frac{dP}{P}+\eta_2\frac{d\varphi}{\varphi}+\eta_3\frac{dr}{r}+\eta_4\frac{dt}{t}+... \quad (4)$$

where t is the duration of static contact between thread and cylindrical surface. The communication potentials are obtained experimentally by the formulae:

$$\eta_1=\frac{\Delta T}{T}:\frac{\Delta P}{P};\varphi=const;\ r=const;t=const \quad (5)$$

$$\eta_2=\frac{\Delta T}{T}:\frac{\Delta\varphi}{\varphi};P=const;\ r=const;t=const \quad (6)$$

$$\eta_3=\frac{\Delta T}{T}:\frac{\Delta R}{R};P=const;\ \varphi=const;t=const \quad (7)$$

$$\eta_4=\frac{\Delta T}{T}:\frac{\Delta t}{t};P=const;\ \varphi=const;r=const \quad (8)$$

In order to find the partial communication potentials $\eta_1,\eta_2,\eta_3,\eta_4$ as per Euler's theoretical model, taking logarithm and differentiation of equation (2) is needed, i. e.:

$$\ell nT=\ell nP+\ell n\left(e^{\mu.\varphi}-1\right)$$

$$\frac{dT}{T}=\frac{dP}{P}+\frac{d\left(e^{\mu.\varphi}-1\right)}{e^{\mu.\varphi}-1}=\frac{dP}{P}+\frac{e^{\mu.\varphi}}{e^{\mu.\varphi}-1}.\mu.\varphi\frac{d\varphi}{\varphi} \quad (9)$$

Comparison of equations (9) and (4) gives:

$$\eta_1=1;\ \eta_2=\mu.\varphi\frac{e^{\mu.\varphi}}{e^{\mu.\varphi}-1};\ \eta_3=0;\eta_4=0 \quad (10)$$

In the particular case when $e^{\mu.\varphi}>>1$, (2) and (10) lead to:

$$T\approx P.e^{\mu.\varphi} \quad (11)$$

$$\eta_2=\mu.\varphi \quad (12)$$

2.2 Experimental determination of the partial communication potentials

To find η_1,η_2,η_3 and η_4 in compliance with formulae (5), (6), (7) and (8) it is necessary to obtain experimentally the laws for the initial friction force T in the form:

$$T_1=T_1(P),\ \varphi=const;\ r=const;t=const \quad (13)$$

$$T_2=T_2(\varphi),\ P=const;\ r=const;t=const \quad (14)$$

$$T_3=T_3(r),\ P=const;\ \varphi=const;t=const \quad (15)$$

$$T_4=T_4(t),\ P=const;\ \varphi=const;r=const \quad (16)$$

Procedure for measuring the initial friction force

As per Figure 1 and Euler's law in the form

$$F=P.e^{\mu.\varphi}$$

as well as having in view that the force F is superimposed by the balancing load P and an additional tension force F^* read by the electronic dynamometer, i.e.

$$F=P+F^*=P+T$$

we obtain

$$T=F^* \quad (17)$$

This means that the friction force T is taken from the electronic dynamometer and is described by Euler's formula (2).

The present work allows Euler's formula to be verified for the initial friction force by means of comparing the theoretically and experimentally determined communication potentials. Because of the huge amount of experimental work, we consider below the communication potentials η_3 and η_4 only.

Determination of communication potential η_3

Initial friction of a textile thread on duralumin multi-stage cylinder with radii r_1, r_2, r_3, r_4, r_5 is being studied (Figure 2).

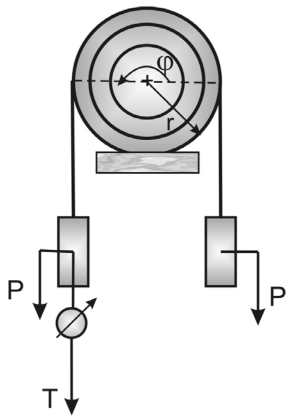

Figure 2. Scheme for the study of the initial friction force of multi-stage cylinder

Figure 3 shows the relationship of the initial friction force T and radius r for constant but different values of P and φ.

It is seen that in all these relations the force T has different values but remains constant and does not depend on r, that is why

$$\eta_3 = \frac{\Delta T_3}{T} : \frac{\Delta r}{r} = 0 \qquad (18)$$

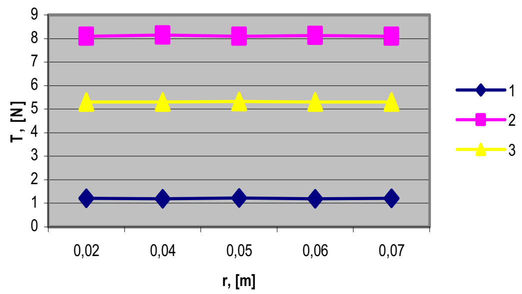

Figure 3. Relationship between initial friction force T and cylinder radius r:

1 - $P = 1{,}3\ N$, $\varphi = \pi$; 2 - $P = 1{,}3\ N$, $\varphi = 3\pi$; 3 - $P = 2{,}0\ N$, $\varphi = 5\pi$.

Determination of communication potential η_4

The reactive communication potential η_4 takes account of the influence of the duration t of the static contact between the thread and the surface upon the initial friction force T; the friction force increases with the duration t of the contact, i.e. $\eta_4 \neq 0$. The influence of t on the force T diminishes with the time.

We assume that this decrease follows the formula in the form:

$$T_4 = T_4^o . t^{k_1}; \ \eta_4 = c_1 t^{\alpha} \qquad (19)$$

and/or the exponential formula:

$$T_4 = T_4^o . e^{-k_{2t}}; \ \eta_4 = c_2 t^{-\beta t} \qquad (20)$$

2.2. Improved forms of Euler's law

The first case of the forms is presented here below. In order to obtain the actual law of initial force T_4 variation, we take the general law of contact interaction under assumption (19), because:

$$\eta_1 = 1; \ \eta_2 = \mu.\varphi; \ \eta_3 = 0; \ \eta_4 = c_1 t^{\alpha} \qquad (21)$$

Substitution of (21) in equation (4)

$$\frac{dT}{T} = \frac{dP}{P} + \mu.\varphi \frac{d\varphi}{\varphi} + c_1 t^{\alpha} \frac{dt}{t} \qquad (22)$$

then integration and taking antilogarithm of (22) lead to:

$$\ell nT = \ell nP + \mu.\varphi + \frac{c_1}{\alpha} t^{\alpha} = \ell nP + \mu.\varphi + c^* t^{\alpha}$$

$$T = P.e^{\mu.\varphi + c^* t^{\alpha}} \qquad (23)$$

Equation (23) represents the law of initial friction force after Euler at large enfold angles $e^{\mu.\varphi} >> 1$.

In the general case, according to (10) we have

$$\eta_2 = \mu.\varphi \frac{e^{\mu.\varphi}}{e^{\mu.\varphi} - 1}$$

Next after substitution in (4), integration and taking antilogarithm we obtain:

$$\frac{dT}{T} = \frac{dP}{P} + \mu.\varphi \frac{e^{\mu.\varphi}}{e^{\mu.\varphi} - 1} \frac{d\varphi}{\varphi} + c_1 t^{\alpha} \frac{dt}{t}$$

$$\ell nT = \ell nP + \ell n\left(e^{\mu.\varphi} - 1\right) + c^* t^{\alpha}$$

$$T = P\left(e^{\mu.\varphi} - 1\right) e^{c^* . t^{\alpha}} \qquad (24)$$

Equation (24) corresponds to the law of initial friction force after Euler in the general case of enfold' angles. The constants c^* and α are experimentally found through measurement of friction force and drawing the relationship $T = T(t)$ for several values of P and φ (See Figure 4).

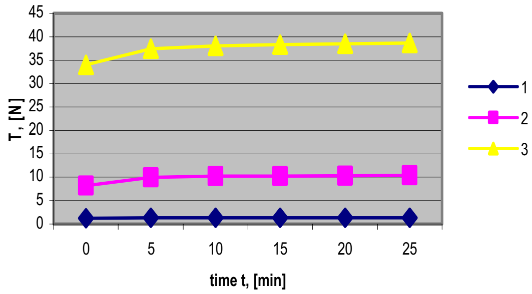

Figure 4. Relationship of friction force T and duration t of the static contact: 1- $P=1{,}3\ N$, $\varphi=\pi$; 2 - $P=1{,}3\ N$, $\varphi=3\pi$; 3 - $P=1{,}3\ N$, $\varphi=5\pi$.

After processing the experimental data, the following values of the constants have been obtained:

$$c^*=0{,}05;\ \ \alpha=0{,}2.$$

The coefficient of friction μ is determined according to Euler's formula in the form (2) by means of measurement of friction force T at the following conditions:

$$\varphi=\pi;\ P=1{,}3\text{ N};\ r_1=20.10^{-3}\text{ m};\ t=0\text{, i.e.}$$

$$\mu=\frac{1}{\varphi}\ell n\frac{P+T}{P} \qquad (25)$$

For the case of that experiment the friction coefficient is $\mu=0{,}21$.

A new moment is that the improved law of initial friction force in its two forms (23) and (24) takes into account the natural influence of contact upon the behavior of tribosystems related to the changes in the real area of interaction throughout time *t*.

3. CONCLUSION

The interdisciplinary paradigm of tribology presupposes availability of a general law of contact interaction. Above paper reveals the putting into operation of that general law in a particular law in the field of tribomechanics, namely Euler's law of friction between flexible element and cylindrical surface.

Following new elements are to be outlined in the present work:

- The classical communication potentials for Euler's law η_1,η_2,η_3 and η_4 are obtained following the universal procedure of the law of contact interaction.
- Experimental procedure has been developed for the study of the initial friction force and of the communication potential η_4 taking into account the effect of the duration of static contact.
- The law of variation of the communication potential η_4 is found and Euler's law on the initial friction force is obtained in two forms taking into account the effect of the duration of static contact.

The proposed procedure allows expanded study of the influence of supplementary external and internal factors on the contact system – humidity, temperature, surface layer roughness, nature of bodies, dynamic impacts, etc.

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