Serbian Tribology Society

SERBIATRIB '11

12th International Conference on Tribology

Faculty of Mechanical Engineering in Kragujevac

Kragujevac, Serbia, 11 – 13 May 2011

MULTICRITERIA OPTIMIZATION OF PANETARY GEAR TRAIN USING EVOLUTIONARY STRATEGIES

B. Rosić[1], Lj. Janković[2], M. Milojević[1]

[1] University of Belgrade, Serbia, Faculty of Mechanical Engineering, e-mail:brosic@mas.bg.ac.rs

[2]Grupa Zastava Vozila, Kosovska 4, Kragujevac, Serbia

Abstract: *Planetary gear trains take a very significant place among the gear transmissions which are used in many branches of industry. This power transmission unit can handle larger torque loads relative to its compact size than any other gear combination in standard transmission. With regard to the growing requirements concerning the economical consumption of energy, the utilization ratio represents a very significant qualitative and quantitative performance of gears. This paper gives the utilization ratio analyze of planetary gear train, starting from kinematics of contacted gears and gear profiles, including sliding and rolling losses resulting from the formation of EHD lubrication, with the numerical results of the instantaneous efficiency of a gear pair with internal gearing. Geometric and operating constraints for internal and external gears and whole planetary gear trains are defined. A method is described to solve a nonlinear parameter optimization problem with several objective functions. For the defined multi-objective optimisation model of the planetary gear train, a computer program based in interactive dialogue is developed. The results showed that the genetic algorithm is useful and applicable for optimization of planetary gears design. The genetic algorithm is an efficient search method which is inspired from natural genetics selection process to explore a given search space.*

Keywords: *planetary gear train, efficiency, sliding, losses, Pareto-optimal, Genetic Algorithm*

1. INTRODUCTION

Planetary gear trains take a very significant place among the gear transmissions which are used in many branches of industry. Planetary gear trains have a number of advantages as compared to the transmission with fixed shafts. Under similar operating conditions the planetary transmissions serve longer and produce less noise compared to the fixed shaft transmissions. This power transmission unit can handle larger torque loads relative to its compact size than any other gear combination in standard transmission.

In articles [1] – [12] authors analyzed similar subject based on planetary gear design. Topics of articles [13] – [14] are related to numerical analyses based upon genetic algorithm and neural networks.

The design of planetary gear trains requires a whole range of geometrical and kinematics conditions in order to perform the mounting and an appropriate meshing of the gears during their work. It is necessary to express the above requirements in terms of the corresponding functional constraints, whereby all the relevant values of the gears and planetary gear trains as a system are defined.

Multi objective optimization techniques generally give a set of compromise solutions, a so-called Pareto-optimal set. The definition of Pareto optimality states that the vector is chosen as optimal if no criterion can be improved without at least one other criterion. Genetic Algorithms are nondeterministic stochastic search/optimization methods that utilize the theories of evolution and natural selection to solve a problem within a complex solution space.

2. PLANETARY GEAR TRAIN EFFICIENCY

According to their kinematics structure, planetary gear trains are complex toothed mechanisms which can be decomposed into external and internal toothed gears with the corresponding interaction. This means that in formulating an optimization model for a planetary gear train, it is necessary, first of all, to define the functional constraints and criteria functions both for the external and internal gears, and then for the planetary gear train as a mechanical system.

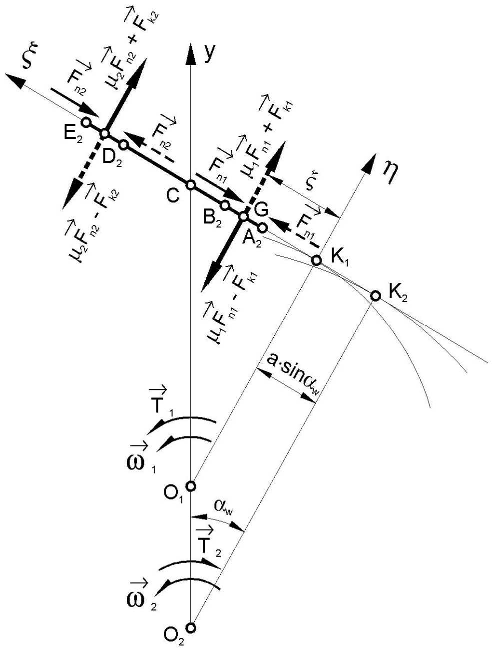

Figure 1. Forces between gear teeth.

In order to ensure the mounting as well as the correct meshing of the gears, it is necessary to fulfill the requirements regarding their alignment, the clearance between the planetary gear trains and their meshing with the sun gear. It is necessary to express the above requirements by the corresponding functional constraints, and based upon them, to identify all relevant values together with the areas of their practical applications.

The analysis considers sliding losses, which are the result of friction forces developed as the teeth slide across each other, rolling losses resulting from the formation of an elasto -hydrodynamic film. The instantaneous efficiency for internal gear at any particular instant, from the relevant T1 input torque, is determined according to the expression:

$$\eta_i = \frac{T_2}{T_1} = \frac{1}{u_{gb}^{H}} \tag{1}$$

Sliding and rolling losses were evaluated by numerically integrating the instantaneous values of these losses across the path of contact. Contact starts at the intersection of the tip diameter of the internal gear with the path of contact at A_2. The path of contact is tangent to the base circles of two gears. Contact ends at the intersection of the tip diameter of the external gear with the path of contact at E_2.

The overall efficiency for gearing under consideration may be written:

$$\eta_{gb}^{H} = \frac{1}{l} = \int_{A_2}^{E_2} \eta_i d\xi \tag{2}$$

The instantaneous frictional force due to sliding of two gear teeth against each other is:

$$F_{\mu}(\xi) = \mu(\xi) F_n K_{\alpha}(\xi) \tag{3}$$

The friction coefficient is calculated by the method of Benedict and Kelley for mineral oil:

$$\mu(\xi) = 0{,}0127 \log\left(\frac{\frac{29{,}66}{b} F_n}{\eta v_{ki} v_{ko}^{2}}\right) \tag{4}$$

The instantaneous force due to build up of the EHD film is

$$F_R = Chb \tag{5}$$

The gear contact minimum film thickness is calculated by the method of Dowson and Higginson

$$h(\xi) = 1{,}6\alpha^{0{,}6} (\eta v_{ko})^{0{,}7} E^{0{,}003} \frac{R^{0{,}43}}{F_n^{0{,}13}} \tag{6}$$

In order to evaluate the efficiency of an internal gear pair it must be considered equilibrium of the gears. Fig. 1 shows the normal forces F_n, the rolling friction forces F_R, and the sliding friction forces F_{μ}, with suffices 1 for teeth in the path of approach and 2 for teeth in the path of recess. One pair of teeth is in contact at point G and the other at point H. For convenience the output torque of the train is assumed constant. From the equilibrium of gears, it comes:

$$F_{n1} = \frac{T_1 - p_1 F_{R2} - \xi F_{R1}}{db_1 + \mu_2 P_1 - \mu_1 \xi} \tag{7}$$

$$T_2 = F_{n1} d_{b2} + p_2(\mu_2 F_{n1} - F_{R2}) - (\mu_1 F_{n1} + F_{R1}) p_3 \tag{8}$$

where

$$p_1 = \xi + p_b \tag{9}$$

$$p_2 = a \sin \alpha_w \xi + x + p_b \tag{10}$$

$$p_3 = a \sin \alpha_w + \xi \tag{11}$$

On the basis of the models developed for a gear pair with external and internal gearing, the efficiency of a planetary gear train may be expressed as:

$$\eta_{aH}^{b} = \frac{1-\eta_{ab}^{H} u_{ab}^{H}}{1-u_{ab}^{H}} \quad (12)$$

where

$$\eta_{ab}^{H} = \eta_{ab}^{H} \eta_{gb}^{H} \quad (13)$$

3. FORMULATION OF OPTIMIZATION MODEL

The criteria regarding the desired performances are expressed by the criteria functions, which, for the best planetary gear train design, should reach the extreme:

$$\underset{x \in D}{extr}\, f(x) \quad (14)$$

The function criteria for a one-stage planetary gear train can be written in the form of the following relations for:

Eq. [15] – [23] are called objective functions. In addition, it is also necessary to include the functional constrains in the form of the inequalities:
Eq. [24] – [31] are called inequality constrains for objective functions.
Eq. [32] – [34] are called equality constrains for objective functions.

Based upon the objective functions given and upon the functional constraints, all the relevant values of the planetary gear train have also been identified.

- centre distance $$f_1 = \frac{m_n z_a}{\cos\beta}(1+u_{a-g}^{H})\frac{\cos\alpha_t}{\cos\alpha_w} \quad (15)$$

- efficiency $$f_2 = \frac{1-\eta_{ab}^{H} u_{ab}^{H}}{1-u_{ab}^{H}} \quad (16)$$

- contact ratio $$f_3 = \varepsilon_{\alpha_{a-g}}(x) \quad (17)$$

- pressure angle $$f_4 = \alpha_{w_{a-g}}(x) \quad (18)$$

- safety factor for bending stress $$f_5 = S_{F1}(x) = \frac{[\sigma_F] M_1}{\sigma_{F1}} \quad (19)$$

- safety factor for contact stress $$f_6 = S_{H1}(x) = \frac{[\sigma_H] M_1}{\sigma_{H1}} \quad (20)$$

- volume of material used for gears $$f_7 = V(x) \quad (21)$$

- safety factor for bending stress $$f_8 = S_{F3}(x) = \frac{[\sigma_F] M_3}{\sigma_{F3}} \quad (22)$$

- outer diameter $$f_9 = D_{out}(x) \quad (23)$$

- bending stress $[\sigma_F]$ $$g_1 = \frac{[\sigma_F] M_1}{\sigma_{F1}} - S_F > 0 \quad (24)$$

$$g_2 = \frac{[\sigma_F] M_2}{\sigma_{F2}} - S_F > 0 \quad (25)$$

$$g_3 = \frac{[\sigma_F] M_3}{\sigma_{F3}} - S_F > 0 \quad (26)$$

- contact stress $[\sigma_H]$ $$g_4 = \frac{[\sigma_H] M_4}{\sigma_{H4}} - S_H > 0 \quad (27)$$

$$g_5 = \frac{[\sigma_H] M_5}{\sigma_{H5}} - S_H > 0 \quad (28)$$

- tip interference $$g_6 = \delta > 0 \quad (29)$$

- radial interference $$g_7 = \Delta x > 0 \quad (30)$$

- space requirements $$g_8 = 2a\sin(\frac{\pi}{n_w}) - f - d_{a-g} \geq 0 \quad (31)$$

- specific sliding $$h_1 = (u_{a-g}^{H})^2 \rho_E \rho_A - \rho_A \rho_E = 0 \quad (32)$$

- shaft alignment $$h_2 = \frac{z_a + z_g}{\cos\alpha_{wb-g}} - \frac{z_b + z_g}{\cos\alpha_{wb-g}} = 0 \quad (33)$$

- condition for assembly $$h_3 = \frac{z_a z_b}{n_w D(z_g z_b)} - INT = 0 \quad (34)$$

4. OPTIMIZATION PROCEDURE USING MULTI OBJECTIVE GENETIC ALGORITHM

GAs is one type of EAs, which was developed by John Holland in the early 1970s. Every genetic algorithm has its basic components shown as the flowchart in Fig. 2. Simple GA has three basic operators: Selection, Crossover, Mutation.

Each member in this population is evaluated and assigned a fitness value. In the selection procedure, some selection criterion is applied to select a certain number of strings, namely parents, from this population. Parent pairs are randomly chosen from the selected population and the kind of merging depends on the crossover operator used.

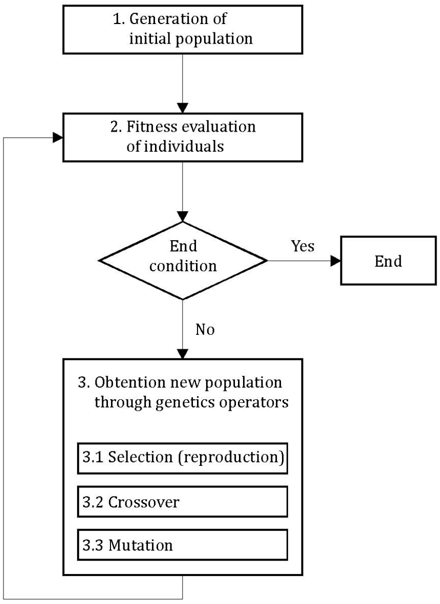

Figure 2. Simple genetic algorithm flowchart.

Mutation simply changes one bit 0 to 1 and vice versa, at a position determined by some rules. Mutation is simple but still important in evolution because it further increases the diversity of the population members and enables the optimization to get out of local optima.

The genetic algorithm (GA) is an optimization and search technique based on the principles of genetics and natural selection. A GA allows a population composed of many individuals to evolve under specified selection rules to a state that maximizes the "fitness" (i.e., minimizes the cost function).

Some of the advantages of a GA include that it
- Optimizes with continuous or discrete variables,
- Doesn't require derivative information,
- Simultaneously searches from a wide sampling of the cost surface,
- Deals with a large number of variables,
- Is well suited for parallel computers,
- Optimizes variables with extremely complex cost surfaces (they can jump out of a local minimum),
- Provides a list of optimum variables, not just a single solution,
- May encode the variables so that the optimization is done with the encoded variables, and
- Works with numerically generated data, experimental data, or analytical functions [15].

The basic idea of the approach is to start with a set of designs, randomly generated using the allowable values for each design variable. Each design is also assigned a fitness value, usually using the cost function for unconstrained problems or the penalty function for constrained problems. From the current set of designs, a subset is selected randomly with a bias allocated to more fit members of the set.

The process is continued until a stopping criterion is met. In the following paragraphs, some details of implementation of these basic steps are presented and explained. First, it shall be defined and explain various terms associated with the algorithm.

5. NUMERICAL RESULTS AND DISCUSSION

In the binary algorithm, after defining the initial population each unknown variable must be encoded and as such must be treated by the end of the process. When the algorithm finds the optimal value, it must be decoded before the end of the process, so that the optimization results was presented as a numerical value. This optimization problem has 8 inequality constraints and 3 equality constraints. Binary coding of specimens is based on the range limits.

Table 1. GA coding of design variables.

Design variables vectors	Vectors	Random binary digits	String length l
Module m_n	x_1	10101	5
Number of teeth z_a	x_2	1101	4
Number of teeth z_g	x_3	11010	5
x_a	x_4	11100	5
x_g	x_5	11100	3
x_b	x_6	111	3
a_1	x_7	11101	5
n_w	x_8	11111	5
H	x_9	101	3
r_{ct}	x_{10}	1111	4
A single 42-bit individual (chromosome)	1010111011101011100111 00 11111101111111011111		

For example, let the only variables be x_1, x_2, x_3. And let the constraints are in the form of equity, for all three variables from range [1,2]. Then, each of the three variables can take only integer values from 1 and 2 and that means that for their binary encoding are sufficient only two binary digits. One of the possible solution to the optimization problem must have all three variables, and individual genetic algorithm has a total of 6 binary digits. For example, if a solution has this formula:

$$x_1 = 0 x_2 = 0 x_3 = 1,$$

in binary, that means 000001.

In this paper, because of nature of the problem and the conditions of the constraints variables x_5, x_6, x_9 are coded with 3 binary digits, x_2, x_{10} are coded with 4 binary digits and $x_{1,} x_{3,} x_4, x_7, x_8$ are coded with 5 binary digits. That means that one possible solution must be coded in 42-digit binary code. The way of coding the variables is shown in the Table 1.

Following parameters are selected to be used for performing operation of genetic algorithm:

-a binary encoding,
-proportional selection,
-crossover around one point,
-the population size of $\mu = 10$ individuals
-the probability of crossing $p_c = 0{,}8$,
-probability of mutation $p_m = 0{,}01$,
-maximum number of generations $g_{\max} = 500$.

Table 2. A sample case at first generation regarding satisfaction of constraints.

First generation	m_n	z_a	z_g	x_a	x_g
36758890.1	1.25	24	70	0.33	0.54
15648796.1	1.45	26	49	0.55	0.54
13376868.7	1.60	25	69	0.67	0.46
23478909.2	5.0	35	56	0.6	0.58
59890097.1	2.0	23	57	0.82	0.52
13478976.8	2.0	23	50	0.25	0.40
65784634.8	2.0	21	56	0.27	0.68
23768965.5	1.5	20	45	0.48	0.62
34897453.5	1.25	10	66	0.70	0.62
5515354.8	1.5	17	60	0.60	0.67
First generation	x_b	a_1	n_w	h	r
36758890.1	0.65	0.603	1	4.22	0.367
15648796.1	0.66	0.620	2	2.46	0.246
13376868.7	0.64	0.614	5	4.66	0.135
23478909.2	0.50	0.601	7	2.56	0.234
59890097.1	0.36	0.604	6	4.70	0.145
13478976.8	0.36	0.602	5	5.52	0.256
65784634.8	0.45	0.606	2	2.54	0.234
23768965.5	0.45	0.607	1	3.2	0.123
34897453.5	0.51	0.601	8	2.7	0.143
5515354.8	0.53	0.620	7	3.80	0.156

Table 3. A sample case at last generation regarding satisfaction of constraints.

Last generation	m_n	z_a	z_g	x_a	x_g
6214.87	2.0	30	54	0.252	0.064
6214.87	2.0	30	54	0.252	0.064
6214.87	2.0	30	54	0.252	0.064
6214.87	2.0	30	54	0.252	0.064
6214.87	2.0	30	54	0.252	0.064
6214.87	2.0	30	54	0.252	0.064
6214.87	2.0	30	54	0.252	0.064
6214.87	2.0	30	54	0.252	0.064
6214.87	2.0	30	54	0.252	0.064
6214.87	2.0	30	54	0.252	0.064

Last generation	x_b	a_1	n_w	h	r
6214.87	0.320	0.57	2	6.67	0.17
6214.87	0.320	0.57	2	6.67	0.17
6214.87	0.320	0.57	2	6.67	0.17
6214.87	0.320	0.57	2	6.67	0.17
6214.87	0.320	0.57	2	6.67	0.17
6214.87	0.320	0.57	2	6.67	0.17
6214.87	0.320	0.57	2	6.67	0.17
6214.87	0.320	0.57	2	6.67	0.17
6214.87	0.320	0.57	2	6.67	0.17
6214.87	0.320	0.57	2	6.67	0.17

After doing all the settings of the genetic algorithm starts process. Criteria to stop the process are achieved or the maximum number of generations or achieved stall time.

Phases of process are binary coded, selection, reproduction, mutation, crossover and migration. The genetic algorithm starts by generating a random initial population. In the example under consideration, each population has 10 individuals and one choice is shown in the table 2. Genetic algorithm stopped in 201th generation, and achieved solution which are given in table 3.

The table 3 contains integer values for 10 variables and appropriate values for objective function for each gene from population in selected last generation.

As it is shown in table 3 each gene from population of 10 genes has the same values for variables and the objective functions.

6. CONCLUSIONS

Multi objective modeling reflects very well the design process in which usually several conflicting objectives have to be satisfied such as the efficiency of planetary gear trains and the distance between centers of sun gear and planetary gear. In the present study, an optimization approach based on Genetic Algorithms is proposed to improve gear performances. Bounding parameters values are very important in GA and directly affects solutions.

Optimization of gear train was accomplished using GA. Results accomplished using GA are less than results that are found using analytical method. Results shown in tables 4 and 5 showed that GA is better method analytical method to obtain gear train minimum.

REFERENCES

[1] J. C. Rawers, J. H. Tylczak, D. E. Alman, Wear Evaluation of High Interstitial Stainless Steels, Tribology transactions, Volume 51, Issue 4, pp. 515 – 525.

[2] Joslyn Hili, Andrew V. Olver, Simon Edwards; Leon Jacobs,Experimental Investigation of Elastohydrodynamic (EHD) Film Thickness Behavior at High Speeds, Tribology transactions, Volume 53, Issue 5, pp. 658 – 666

[3] Faruk M., Tamer B., Kurtulus B., Fatih E. B. Optimization of module, shaft diameter and rolling bearing for spur gear through genetic algorithm. Expert Systems with Applications; 37(12):8058-8064; 2010.

[4] Arora J.S. Introduction to optimum design. McGraw-Hill; 1989.

[5] A.Marinković, B.Rosić, G. Petropoulos: Analysis and optimization of dynamically loaded porous metal sliding bearings under conditions of elastohzdrodynamic lubrication,International Jounal for Computer-Aided Engineering and Software, Vol.24 No.3,2007.

[6] Rosić B.: Multicriterion Optimization of Multistage Gear Train Transmission, The Scientific Journal FACTA UNIVERSITATIS, Series: Mechanical Engineering, Vol. 1, No 8, 2001.

[7] A. Marinković, B. Rosić, M. Janković: Optimum Design for Porous Metal Bearing, International Journal of Applied Mechanics and Engineering, 2001, Volume 6, Number 1, Technical University Press Zielona Gora, Poland.5.

[8] Rosić B.: Multicriterion optimization of planetary gear train, Proceedings of the International Gearing Conference, Newcastle, 1994, (195-199).

[9] Ristivojević M., Rosić B., Laćarac V.: The influence of gearing parameters on damaging of gear flanks, Proceedings of the abstracts of the papers from the World Tribology Congress, London, England, 1997, page 763.

[10] Rosić B.: An automatic design optimization procedure for compact gear sets, Proceedings of the International conference on engineering design, Dubrovnik, august 1990., Volume 4 (1896 – 1889).

[11] Vulić A., Rosić B.: The analysis of the factors influencing the harmonic transmitters utilization, Proceedings of the International Conference on Tribology and EUREKA Brokerage Event, Thessaloniki, Greece, 1996., (471-477).

[12] Rosić B., A. Marinković, D. Urošević: Dynamic parameters Analysis of planetary transmission structure, The 2nd International conference “Power Transmissions ’06” Novi Sad, Serbia, 2006., (159-162).

[13] Tripathi V.K, Chauhan H.M. Multi objective optimization of planetary gear train. Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics); 6457:578-582; 2010.

[14] Bonori G, Barbieri M, Pellicano F. Optimum profile modifications of spur gears by means of genetic algorithms. Journal of Sound and Vibration; 313(3-5):603-616; 2008.

[15] Haupt R.L, Haupt S.E. Practical Genetic Algorithms. John Wiley and Sons, INC. Publication, 2004.