SERBIATRIB '11

Serbian Tribology Society

12th International Conference on Tribology

Faculty of Mechanical Engineering in Kragujevac

Kragujevac, Serbia, 11 – 13 May 2011

EFFECT OF CHANGES OF VISCOSITY OF MINERAL OIL IN THE FUNCTION OF PRESSURE ON FLOWING THROUGH A LONG RADIAL CLEARANCE

Darko Knežević[1], Aleksandar Milašinović[1], Zdravko Milovanović[1], Vladimir Savić[2]
[1]Faculty of Mechanical Engineering Banja Luka, BiH – RS
darkokn@blic.net; acom@blic.net, mzdravko@urc.rs.ba,
[2]Faculty of Technical Science Novi Sad, Serbia; savicv@uns.ac.rs

Abstract: *Radial clearance in hydraulic components (but also in motor vehicles, etc) has multiple roles: it allows relative movement of elements to achieve the given functions, provides the sealing function between the space with different levels of pressure, and also serves as hydrostatic or hydrodynamic bearing according to the forces within the component. Common values of radial clearance is approximate in the range from 1 μm (minimum values for servo valves) to 50 μm (maximum value for axial piston pumps).*
It is common that when calculating the flow rate of mineral oil through clearances, constructors calculate with a constant value of viscosity for given working temperature. In this paper is analyzed the error because of acceptance of such assumptions.

Keywords: *pressure, viscosity, radial clearance, overlap, volumetric flow rate*

1. INTRODUCTION

Mineral oils are the most common type fluid used in the hydraulic systems (over 90% of the total use). To be able to achieve the projected functions, in the hydraulic systems are used many control and regulatory components in which they appear radial clearances.

In the analysis of the hydraulic systems are usually not taken into account change of viscosity of mineral oil with a change of pressure, because that change is far smaller than change of viscosity with changes in temperature. However, as the work of well-designed hydraulic system usually takes place at approximately constant operating temperature of working fluid, this neglect may lead to significant errors in calculations.

In practical calculations, it is common that the volumetric flow rate of fluid through radial clearance is calculated using the following formula

$$Q = \frac{d\pi c_r^3 \Delta p}{12\mu_0 L}, \qquad (1)$$

where:
d – diameter;
c_r – radial clearance;
Δp – pressure drop;
μ_0 – dynamic viscosity at the atmospheric pressure;
L – length of overlap.

Experimental measurements show that the volumetric flow rate through radial clearance calculated by formula (1) is differed from the measured values.

2. PRESSURE DEPENDENCE OF VISCOSITY OF MINERAL OIL

Viscosity of oil is increased with growth of pressure. Chemical composition greatly influences on viscosity-pressure characteristics of a hydraulic fluids.

The best known equation, which describes viscosity-pressure behavior of mineral hydraulic fluids, is Barus equation: [1]

$$\mu = \mu_0 e^{\alpha p}, \qquad (2)$$

where:
μ – dynamic viscosity at the pressure 'p' [Pas];

μ_0 – dynamic viscosity at the atmospheric pressure [Pas];

α - pressure-viscosity coefficient, which dependence of pressure and temperature [1/Pas].

To adopt experimental data by a mathematical model, the so-called "Modulus Equation" was used. "Modulus Equation" is based on Barus equation. The model comprises the pressure, p [bar], and temperature, T [^{o}C], dependence of the dynamic viscosity [2]

$$\mu(p,T) = \mu_0 e^{\left[\frac{p}{a_1+a_2T+(b_1+b_2T)p}\right]}. \quad (3)$$

Dependence pressure-viscosity coefficient, α, of pressure and temperature is given by equation

$$\alpha(p,T) = \frac{\ln\mu - \ln\mu_0}{p - p_a} = \frac{1}{a_1 + a_2T + (b_1 + b_2T)p}. \quad (4)$$

The parameters a_1, a_2, b_1, b_2 represent the oil behavior and have to be calculated from experimental data. In accordance with the data given by the mineral oil producers ([3]), by using method of identification unknown parameters of mathematical model, are calculated constants from equation (4).

Table 1. Parameter values for pressure-viscosity coefficient, α [4]

	a_1 [bar]	a_2 [bar/^{o}C]	b_1	b_2 [1/^{o}C]
Mineral oil of paraffinic base structure	334	3.2557	0.026266	0.000315

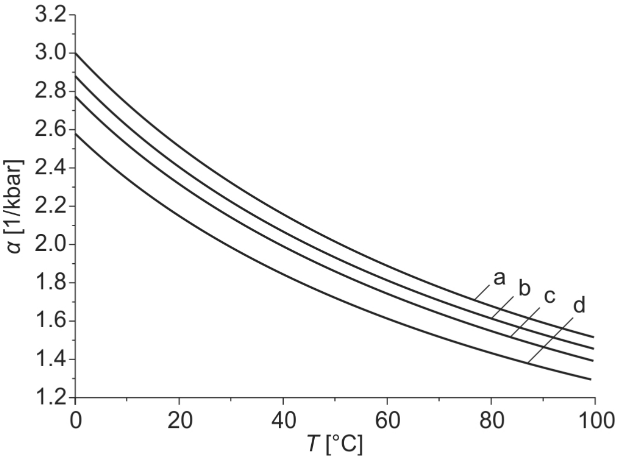

Figure 1. Pressure-viscosity coefficient of mineral oil paraffinic base structure: a) $p = p_a$; b) p = 500 bar; c) p = 1000 bar; d) p = 2000 bar

Example:

For the pressure of 350 bar and temperatures of 20.5 and 40^{o}C, values of pressure-viscosity coefficient, are calculated by using of formula (4):

$$\alpha\left(p = 350\ \text{bar},\ T = 20.5^{o}\ \text{C}\right) = 0.002426\ \text{bar}^{-1}$$

and

$$\alpha\left(p = 350\ \text{bar},\ T = 40^{o}\ \text{C}\right) = 0.0020928\ \text{bar}^{-1},$$

and values of dynamic viscosity, are calculated by using of formula (3):

$$\mu\left(p = 350\ \text{bar},\ T = 20.5^{o}\ \text{C}\right) = 2.3375 \cdot \mu_0$$

and

$$\mu\left(p = 350\ \text{bar},\ T = 40^{o}\ \text{C}\right) = 2.08 \cdot \mu_0.$$

Value of dynamic viscosity, at pressure of 350 bar and temperature of 20.5^{o}C (40^{o}C), is for 2.3375 (2.08) times higher then value of dynamic viscosity at atmospheric pressure and the same temperature.

Figure 2 shows values of dynamic viscosity of some hydraulic oil at 50 ^{o}C, for different values of working pressure.

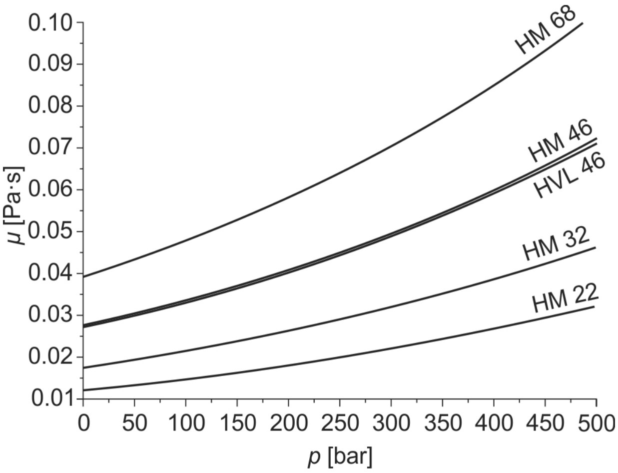

Figure 2. Change of viscosity of some mineral hydraulic oils with the change of pressure (temperature 50 ^{o}C) [4]

3. FLOW OF MINERAL OIL THROUGH LONG RADIAL CLEARANCES

Under the long clearance is understood such clearance whose length overlap (the length of leakage path) L is such that the hydraulic fluid is slightly warmed at flowing through the clearance. [4]

In this case, change of state of mineral oil at flowing can be considered isothermal. Such cases flow through the clearance is processed in the literature, but with one flaw: they ignore change of viscosity of oil with a change of pressure. This neglect can lead to the calculated flow rate is up to 50% higher than the actual flow rate (depending on value of working pressure in the hydraulic system).

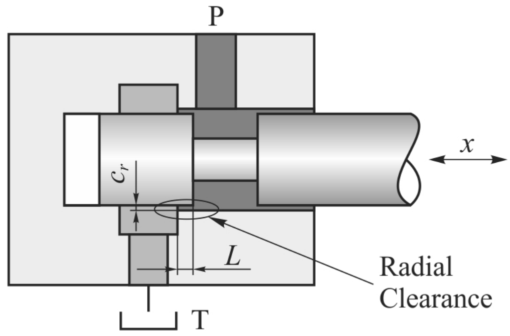

Figure 2. Flow of hydraulic fluid through long radial clearance

The flow of fluid through radial clearance can be described by the Navier-Stokes equations.

For steady flow of incompressible fluid, taking into account the change of viscosity along clearance, the vector form of these equations is

$$\rho \frac{D\mathbf{v}}{Dt} = \rho \mathbf{f} - \mathbf{grad}p + \Delta\left(\mu \mathbf{v}\right). \tag{5}$$

where:

$\rho \frac{D\mathbf{v}}{Dt}$ - inertial force per unit volume on elementary particle of fluid;

$\mathbf{grad}p$ - pressure force per unit volume on elementary particle of fluid,

$\Delta\left(\mu \mathbf{v}\right)$ - viscous force per unit volume on elementary particle of fluid.

Following analysis will be carried out for symmetrical radial clearance because of in real conditions at flowing of oil through the radial clearance within the hydraulic component, pressure seeks to hold piston in concentric (coaxial) position in relation to the cylinder.

Cross sectional area through which fluid flows through the clearance is

$$A = \frac{\left(d_2^2 - d_1^2\right)\pi}{4}, \tag{6}$$

where:

d_2 - inner diameter of cylinder,

d_1 - diameter of piston.

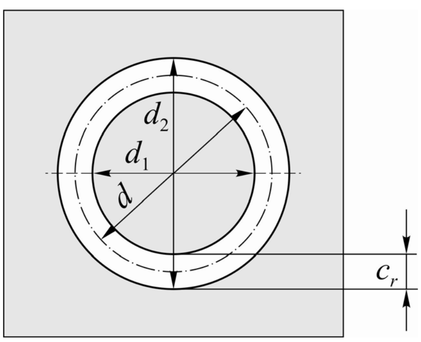

Figure 3. Symmetric radial clearance

As trajectories of fluid particles straight and parallel, fluid flow through radial clearance can be seen as flow between flat parallel surfaces which width are L, and the distance between them is c_r.

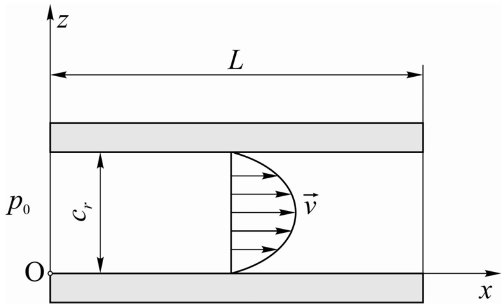

Figure 4. The flow of fluid between parallel plates

Can be written

$$d_1 = d - c_r, \tag{7}$$

and

$$d_2 = d + c_r. \tag{8}$$

Substituting equations (7) and (8) in equation (6), we get

$$A = d\pi c_r = wc_r, \tag{9}$$

where w is defined as the width of rectangular opening.

During steady rectilinear parallel flow there is only longitudinal component of velocity u, while components v and w are equal zero. Profile of velocity for this case of flow does not change along clearance.

Continuity equation in differential form is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \tag{10}$$

As v and w equal zero, from equation (10) follows $\partial u/\partial x = 0$, why $u = f(z)$.

Since the influence of mass forces (**f**) are negligible, the Navier-Stokes equations are reduced to form

$$\frac{dp}{dx} = \frac{d^2\left(\mu u\right)}{dz^2}. \tag{11}$$

Dynamic viscosity of fluid that flowing through clearance, for the specific operating tepmerature, is given by the Barus's equation (2).

As pressure is function of longitudinal coordinate x, and the dynamic viscosity depends on pressure, then the dynamic viscosity is function of coordinates x, so we can write

$$e^{-\alpha p}\frac{dp}{dx}=\mu_0\frac{d^2u}{dz^2}. \qquad (12)$$

As the left side of equation (12) depends only of x and the right side equation (12) depends only of z, it follows that both sides of the equation (12) must be equal to a constant C.

General solution of equation (12) is

$$u=\frac{C}{2\mu_0}z^2+C_1z+C_2. \qquad (13)$$

The constant C can be determined from the boundary conditions for pressure:

$$\text{for } x=0, \qquad p=p_0, \qquad (14a)$$

and

$$\text{for } x=L, \qquad p=0. \qquad (14b)$$

From the condition that the left side of equation (12) is equal to constant C, ie.

$$e^{-\alpha p}\frac{dp}{dx}=C, \qquad (15)$$

putting

$$e^{-\alpha p}\frac{dp}{dx}=-\frac{1}{\alpha}\frac{d}{dx}e^{-\alpha p}. \qquad (16)$$

equation (15) can be directly integrated, and we get

$$e^{-\alpha p}=-\alpha Cx-\alpha C_3. \qquad (17)$$

Incorporating the boundary conditions (14) in equation (17), we get

$$C_3=-\frac{1}{\alpha}e^{-\alpha p_0}, \qquad (18)$$

and

$$C=\frac{e^{-\alpha p_0}-1}{\alpha L}. \qquad (19)$$

Using equations (16) and (19), from equation (15), we can get the law of change of pressure along clearance

$$p=-\frac{1}{\alpha}\ln\left(e^{-\alpha p_0}+\frac{1-e^{-\alpha p_0}}{L}x\right). \qquad (20)$$

As an example, in Figure 5 is shown the change of pressure through the clearance that length of overlap is 10 mm and working pressure 350 bar. In Figure 5 is given an overview of changes of pressure through the clearance: at neglect of influence of pressure on the value of dynamic viscosity, and curves of pressure change which are calculated by formula (20), for different operating temperatures (20°C and 50°C). The working fluid is a mineral hydraulic oil HM 46.

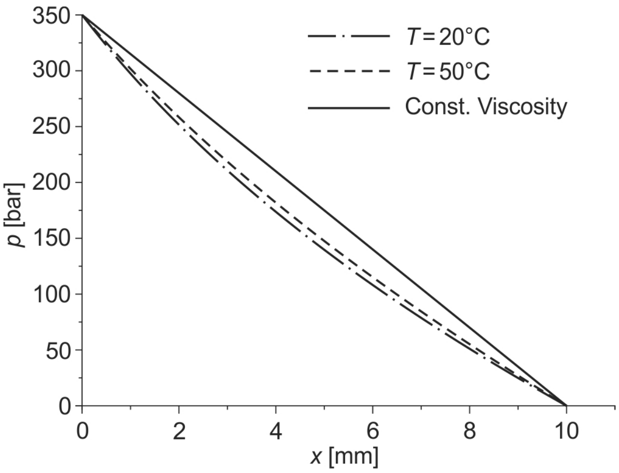

Figure 5. Change of pressure along the clearance

The boundary conditions for velocity are:

$$\text{for } z=0, \qquad u=0, \qquad (21a)$$

and

$$\text{for } z=c_r, \qquad u=0. \qquad (21b)$$

Substituting conditions (21) in equation (13), we get

$$C_2=0, \qquad (22)$$

and

$$C_1=-\frac{C}{2\mu_0}c_r. \qquad (23)$$

Incorporating the constants C_1 and C_2 in equation (13), we get expression for the velocity profile in the cross section of clearance

$$u=\frac{1-e^{-\alpha p_0}}{2\alpha\mu_0 L}\left(c_r z-z^2\right). \qquad (24)$$

Volumetric flow rate through clearance is

$$Q=\frac{1-e^{-\alpha p_0}}{\alpha}\cdot\frac{d\pi c_r^3}{12\mu_0 L}. \qquad (25)$$

4. ORIENTATION (BASED ON RESULTS OF EXPERIMENTS) CRITERIA FOR THE DIVISION CLEARANCES BETWEEN SHORT AND LONG CLEARANCES

Based on the experimental results, it was determined border length L_g for application of the formula (25) [4]

$$L_g=-\lambda\frac{c_r}{\ln 0.9}=9.5\lambda c_r, \qquad (26)$$

where:
λ - coefficient determined from experimental data.

For the lengths of overlap L that are greater than L_g, flow rate through the radial clearance is calculated using the formula (25).

Coeficient λ can be determined from expression ([4]):

$$\lambda = \frac{\sqrt{p_0}}{25 \cdot \beta}. \tag{27}$$

Viscosity-temperature coefficient β can be calculated from equation ([4])

$$\beta = \frac{1}{\Delta T} \ln \frac{\mu_0}{\mu}. \tag{28}$$

where

- $\Delta T = T - T_0$ [°C] - temperature increase,
- μ and μ_0 - values of dynamic viscosity at temperature T and T_0, at atmospheric pressure,
- p_0 – working pressure.

In Figure 6 are given the value of the coefficient β for some oils used in experimental research. [4]

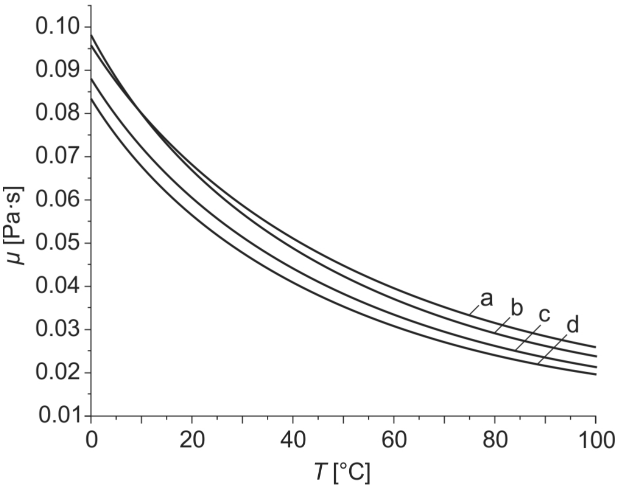

Figure 6. Temperature-viscosity coefficient of hydraulic oil: a) HM 68; b) HM 46; c) HM 32; d) HM 22

5. EXPERIMENTAL VERIFICATION OF THEORETICAL MODEL

In this paper is given an example of check theoretical model at measurement of flow rate through the radial clearance size $c_r = 17$ μm and diameter of the piston $d = 10$ mm. Working pressure was 350 bar. It was used mineral oil HM 46 [4]

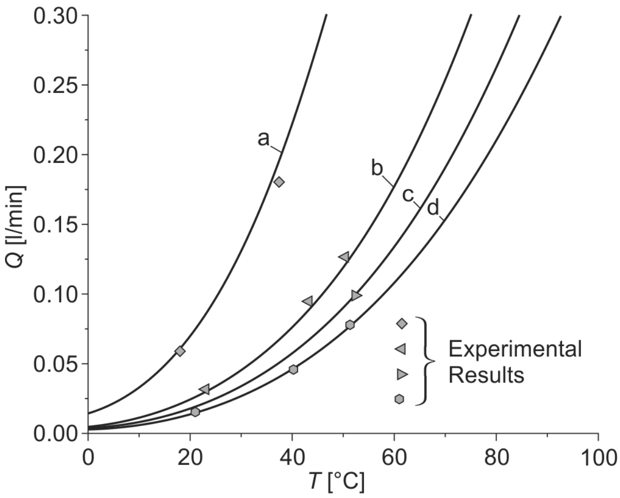

Figure 7. Theoretical curves: a) overlap 2 mm, b) overlap 6 mm; a) overlap 8 mm; a) overlap 10 mm

Table 2. The numerical values of the experimental results in Figure 7 and corresponding values calculated by the theoretical model (Equation 25)

L [mm]	T [°C]	Volumetric flow rate [l/min]	
		Experiment	Theoretical model
2	18	0.058	0.06
	37	0.18	0.19
6	23	0.031	0.028
	43	0.0936	0.087
	50	0.125	0.12
8	52	0.098	0.098
10	21	0.015	0.0148
	40	0.045	0.045
	51	0.0767	0.075

It should be noted that this example provides an illustration. The measuring of the flow rate was carried out for different diameters of pistons, various sizes of radial clearance and various kinds of mineral oils. Results of measurements were showed good agreement between experimental results and those obtained using the theoretical model.

6. CONCLUSION

As the efficiency of components with non-contact sealing clearances depends on the construction, types and characteristics of working fluid, and working pressure and temperature, the results of this study have the following scientific contributions:

- In this paper is presented a detail description of change of viscosity in function of change of pressure for mineral oils (especially mineral hydraulic oils). It is shown that the precise mathematical modeling of the fluid properties are necessary for research phenomena within components in which there are radial clearances;

- It is given mathematical model of flow through long radial clearances, taking into account change of viscosity mineral oil with change of pressure, for given working temperature;
- The derived formulas can be applied for flow through radial clearances with the lengths of overlap that are greater than those given formula (26), and for radial clearances whose size is larger than 10 μm. For radial clearances whose size is less than 10 μm, there is the effect of obliteration, which is not analyzed in this paper.

REFERENCES

[1] JUGOMA – Group of Authors, Lubricants and Lubrication, The Union of Yugoslav Societies for Application of Fuels and Lubricants, Zagreb, 1986.

[2] Schmidt A., Viscosity-Pressure-Temperature Behaviour of Mineral and Synthetic Oils, 12th International Colloquium Tribology 2000., Stuttgart, Germany, 2000.

[3] Frauenstein M., Guide for Fluid Under Pressure for Constructors, Mobil Oil, 1979.

[4] Kneževic D., PhD Thesis, Influence Clearances Geometry in Hydraulic Components of Automation Control on Efficiency of Hydraulic Systems, Novi Sad, 2007.