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LOW REYNOLDS NUMBER NON-ISOTHERMAL MICROBEARING GAS FLOW

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Abstract: *A low Reynolds number non-isothermal compressible gas flow in slider microbearing is investigated in this paper. The gas flow is defined by continuity, Navier-Stokes and energy continuum equations along with the velocity slip and temperature jump boundary conditions at the walls. The dynamic viscosity and the thermal conductivity are assumed to be dependent on temperature. Results are obtained for the slip and continuum flow conditions where the Knudsen number is $Kn \leq 0.1$. The solution is achieved by perturbation method and two approximations are found out. The first approximation corresponds to the continuum flow conditions, while the second one represents the rarefaction effects. As the microbearing height at the exit h_e is much smaller than its length l, the small parameter is defined as $\varepsilon = h_e / l$. Moreover, low Mach number gas flow is assumed. The correlation between the small parameter ε and Knudsen, Reynolds and Mach number enables the precise estimation of each terms contribution in the non-dimensional governing equations and the boundary conditions. The obtained results allow the analysis of the influence of bearing number, wall inclination, walls temperature difference as well as Knudsen number on the bearing capacity.*

Keywords: *analytical solution, non-isothermal, slip flow, rarefied gas, microbearing*

1. INTRODUCTION

Gas flow in the slip regime ($0.001 < Kn < 0.1$) in microbearings is frequently present in micro-electro-mechanical systems (MEMS) [1,2]. There are several different solutions for isothermal microbearing gas flow in literature [3-7].

In this paper a low Reynolds number rarefied non-isothermal compressible gas flow in slider microbearing is investigated [8-10]. An analytical solution is obtained by continuity, Navier-Stokes and energy continuum equations along with the velocity slip and temperature jump boundary conditions at the walls. The dynamic viscosity and thermal conductivity are assumed to be dependent on temperature.

2. PROBLEM DESCRIPTION

In the Figure 1 a two-dimensional non-isothermal compressible gas flow in microbearing with different temperatures of the walls is depicted. A small parameter is defined as the ratio between exit microbearing height and microbearing length: $\varepsilon = \tilde{h}_e / \tilde{l}$.

The sign ~ is used for dimensional quantities, while non-dimensional quantities are without that. It is assumed that the channel height is slowly varying, so the cross-wise velocity component is much smaller than the stream wise component ($\tilde{v} = \varepsilon \tilde{V}$, $\tilde{V} = O(1)$). The slip gas is subsonic, so the Knudsen, Mach number and the ratio between Mach and Reynolds number is taken to be of the order of a small parameter ε:

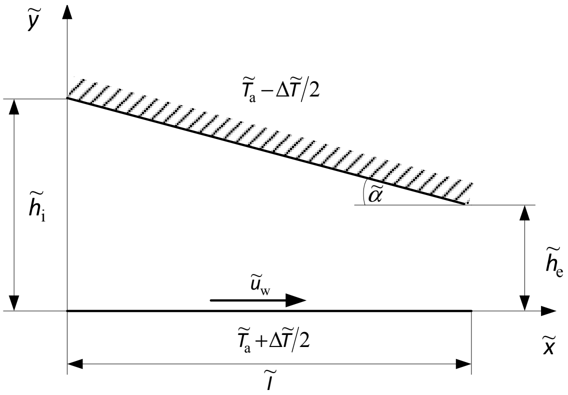

Figure 1. Slider microbearing with different walls' temperatures

$$Kn_{\mathrm{r}} = \eta\, \varepsilon^{n},\ \kappa\, Ma_{\mathrm{r}}^{2} = \beta\, \varepsilon^{m},\ \frac{\kappa\, Ma_{\mathrm{r}}^{2}}{Re_{\mathrm{r}}} = \gamma\, \varepsilon\,. \quad (1)$$

where $m > 0, n > 0$, where $\eta = O(1)$, $\beta = O(1)$ and $\gamma = O(1)$. Kn_r, Ma_r and Re_r are reference Mach, Reynolds and Knudsen numbers defined as $Ma_{\mathrm{r}} = \tilde{u}_{w} \big/ \sqrt{\kappa \tilde{R} \tilde{T}_{\mathrm{a}}}$, $Kn_{\mathrm{r}} = \tilde{\lambda}_{\mathrm{r}} / \tilde{h}_{\mathrm{e}}$, $Re_{\mathrm{r}} = \dfrac{\tilde{u}_{\mathrm{w}} \tilde{h}_{\mathrm{e}} \tilde{p}_{\mathrm{e}}}{\tilde{\mu}_{\mathrm{r}} \tilde{R} \tilde{T}_{\mathrm{a}}}$, where $\tilde{u}_{w}$ is wall velocity, $\tilde{T}_{\mathrm{a}} = \left(\tilde{T}_{\mathrm{w}1} + \tilde{T}_{\mathrm{w}2} \right) / 2$ is the average temperature of the walls, $\tilde{\lambda}_{\mathrm{r}} = \tilde{\mu}_{\mathrm{r}} \sqrt{\pi R \tilde{T}_{\mathrm{a}} / 2} \big/ \tilde{p}_{\mathrm{e}}$ is the reference molecular mean-free path, $\tilde{\mu}_{\mathrm{r}}$ is the reference dynamic viscosity, $\tilde{p}_{\mathrm{e}}$ is the pressure at the exit, $\tilde{h}_{\mathrm{e}}$ is the microbearing height at the exit and R is the gas constant. According to (1) the order of reference Reynolds number is $Re_{\mathrm{r}} = \dfrac{\beta}{\gamma} \varepsilon^{m-1}$. Also, from the well known relation between Kn_{r} , Ma_{r} , and Re_{r} and (1) follow $2n + m = 2$ and $\beta = \gamma^{2} \pi / \left(2\eta^{2} \right)$. The parameter γ is in relation with the bearing number $\Lambda = 6 \tilde{\mu}_{\mathrm{r}}\, \tilde{u}_{\mathrm{w}} \tilde{l} \big/ \left(\tilde{p}_{\mathrm{e}} \tilde{h}_{\mathrm{e}}^{2} \right)$ as $\gamma = \Lambda / 6$.

Now, neglecting the terms which order is $O(\varepsilon)$ or lower, the continuity equation, the Navier-Stokes equations, the energy equation and the equation of state in the dimensionless form follow:

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho V)}{\partial y} = 0\,, \qquad (2)$$

$$\frac{\partial p}{\partial x} = \frac{\Lambda}{6} \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right), \qquad (3)$$

$$\frac{\partial p}{\partial y} = 0\,, \qquad (4)$$

$$\frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) = 0\,, \qquad (5)$$

$$p = \rho T\,, \qquad (6)$$

where non-dimensional quantities are $x = \tilde{x} / \tilde{l}$, $y = \tilde{y} / \tilde{h}_{\mathrm{e}}$, $u = \tilde{u} / \tilde{u}_{\mathrm{w}}$, $V = \tilde{V} / \tilde{u}_{\mathrm{w}}$, $T = \tilde{T} / \tilde{T}_{\mathrm{a}}$, $p = \tilde{p} / \tilde{p}_{\mathrm{e}}$, $\mu = k = T^{a}$, $\rho = \tilde{\rho} \tilde{R} \tilde{T}_{\mathrm{a}} / \tilde{p}_{\mathrm{e}}$ and k is thermal conductivity.

Maxwell-Smoluchowski first-order slip boundary conditions in the dimensionless form determine the gas velocity and the temperature at the walls:

$$\begin{aligned} & y = 0 : u = 1 + \frac{2 - \sigma_{v}}{\sigma_{v}} Kn_{\mathrm{r}} \frac{T^{(0{,}5+a)}}{p} \frac{\partial u}{\partial y}, V = 0, \\ & y = h(x) : u = -\frac{2 - \sigma_{v}}{\sigma_{v}} Kn_{\mathrm{r}} \frac{T^{(0{,}5+a)}}{p} \frac{\partial u}{\partial y}, V = u \frac{dh(x)}{dx}, \end{aligned} \qquad (7)$$

$$\begin{aligned} & y = 0 : T = 1 + \theta + \frac{2 - \sigma_{T}}{\sigma_{T}} \left(\frac{2\kappa}{\kappa + 1} \right) \frac{Kn_{\mathrm{r}}}{Pr} \frac{T^{(0{,}5+a)}}{p} \frac{\partial T}{\partial y}, \\ & y = h(x) : T = 1 - \theta - \frac{2 - \sigma_{T}}{\sigma_{T}} \left(\frac{2\kappa}{\kappa + 1} \right) \frac{Kn_{\mathrm{r}}}{Pr} \frac{T^{(0{,}5+a)}}{p} \frac{\partial T}{\partial y}, \end{aligned} \qquad (8)$$

where θ is defined as $\theta = \dfrac{\Delta T}{2} = \dfrac{\tilde{T}_{w1} - \tilde{T}_{w2}}{\tilde{T}_{w1} + \tilde{T}_{w2}}$.

The solutions are obtained by perturbation method. All dependant variables from equations (2)-(6) i.e. pressure, temperature and velocity components are assumed by two terms of perturbation series:

$$f = f_{0} + \frac{Kn_{\mathrm{r}}}{\eta} f_{1} \qquad (9)$$

where f_{0} is the solution for continuum, and f_{1} is correction for the slip.

The governing equations and boundary conditions for the first approximation are:

$$\frac{\partial \left(p_{0} u_{0} / T_{0} \right)}{\partial x} + \frac{\partial \left(p_{0} V_{0} / T_{0} \right)}{\partial y} = 0\,, \qquad (10)$$

$$\frac{\partial}{\partial y}\left(T_0^a\frac{\partial u_0}{\partial y}\right)=\frac{1}{\gamma}\frac{\mathrm{d}p_0}{\mathrm{d}x}\,, \tag{11}$$

$$\frac{\partial}{\partial y}\left(T_0^a\frac{\partial T_0}{\partial y}\right)=0\,, \tag{12}$$

$$y=0:\quad u_0=1, V_0=0, T_0=T_{w1}=1+\theta\,, \tag{13}$$

$$y=h:\quad u_0=0, V_0=0, T_0=T_{w2}=1-\theta\,. \tag{14}$$

The first approximation of the temperature is derived from the energy equation (12). Then, the approximation of the velocity is derived from the momentum equation (11). Finally, the pressure approximation follows from the continuity equation (10). The solutions for the first approximation are:

$$T_0=\left[(a+1)(C_1y+C_2)\right]^{\left(\frac{1}{a+1}\right)}, \tag{15}$$

$$u_0=bT_0^{(a+2)}+C_3T_0+C_4\,, \tag{16}$$

$$V_0=cT_0^{(2a+3)}+dT_0^{(a+2)}+eT_0^{(a+1)}+gT_0+i\,, \tag{17}$$

$$\left(h^3p_0p_0'\right)'+C_c\left(h\,p_0\right)'=0\,, \tag{18}$$

where C_2 is constant and C_1, C_3, C_4, C_c, b, c, d, e, g, and i are the functions of x.

The governing equations and boundary conditions for the second approximation are:

$$\frac{\partial}{\partial x}\int_{1+\theta}^{1-\theta}\left(p_0u_1T_0^{a-1}-p_0T_1u_0T_0^{a-2}+p_1u_0T_0^{a-1}\right)\mathrm{d}T_0=0, \tag{19}$$

$$p_1'=\frac{\gamma C_1^2}{T_0^a}\frac{\partial}{\partial T_0}\left(\frac{\partial u_1}{\partial T_0}+a\frac{T_1}{T_0}\frac{\partial u_0}{\partial T_0}\right), \tag{20}$$

$$\frac{\partial}{\partial T_0}\left(a\frac{T_1}{T_0}+\frac{\partial T_1}{\partial T_0}\right)=0\,, \tag{21}$$

$$y=0: u_1=\frac{2-\sigma_v}{\sigma_v}\eta T_{w1}^{0,5}\frac{C_1}{p_0}\left((a+2)T_{w1}^{(a+1)}b+C_3\right),$$
$$V_1=0, T_1=\frac{2-\sigma_T}{\sigma_T}\left(\frac{2\kappa}{\kappa+1}\right)\frac{\eta}{\mathrm{Pr}}T_{w1}^{0,5}\frac{C_1}{p_0}, \tag{22}$$

$$y=h: u_1=-\frac{2-\sigma_v}{\sigma_v}\eta\, T_{w2}^{0,5}\frac{C_1}{p_0}\left((a+2)T_{w2}^{(a+1)}b+C_3\right),$$
$$V_1=u_1h', T_1=-\frac{2-\sigma_T}{\sigma_T}\left(\frac{2\kappa}{\kappa+1}\right)\frac{\eta}{\mathrm{Pr}}T_{w2}^{0,5}\frac{C_1}{p_0}. \tag{23}$$

The solution procedure for the second system of equations is the same as for the first. The solutions for the second approximation are:

$$T_1=C_5^*T_0+C_6^*T_0^{-a}\,, \tag{24}$$

$$u_1=T_0^{(a+2)}\left(b_cp_1'h^2-a\,bC_5^*\right)+ +C_7^*T_0+C_3C_6^*T_0^{-a}+C_8^*\,, \tag{25}$$

$$\left(\frac{p_0p_1'}{C_1^3}\right)'b_cC_{c1}^3\left(\frac{\left(T_{w2}^{(a+1)}+T_{w1}^{(a+1)}\right)}{2}+\frac{C_{c3}}{b_c}-\frac{(a+1)T_{w1}T_{w2}\left(T_{w2}^a-T_{w1}^a\right)}{a\left(T_{w1}-T_{w2}\right)}\right)+C_{c1}\left(\frac{C_3p_1}{C_1}\right)'$$
$$+\left(\frac{b\,p_1}{C_1}\right)'\frac{T_{w2}^{(2a+2)}-T_{w1}^{(2a+2)}}{2(a+1)}+\left[\frac{p_0}{C_1}\left(C_{88}^*-C_4C_5^*\right)\right]'\frac{T_{w2}^a-T_{w1}^a}{a}$$
$$-\left[\frac{(a+1)bC_5^*p_0}{C_1}\right]'\frac{T_{w2}^{(2a+2)}-T_{w1}^{(2a+2)}}{2(a+1)}+$$
$$+\left(\frac{C_4C_6^*p_0}{C_1}\right)'\left(T_{w2}{}^{-1}-T_{w1}{}^{-1}\right)+\left(\frac{C_4p_1}{C_1}\right)'\frac{T_{w2}^a-T_{w1}^a}{a}$$
$$+C_{c1}\left[\frac{p_0}{C_1}\left(C_{77}^*-bC_6^*-C_3C_5^*\right)\right]'=0\,, \tag{26}$$

where C_5^*, C_6^*, C_7^*, C_8^*, C_{77}^* and C_{88}^* are the functions of x.

3. RESULTS AND DISCUSIONS

According to the solutions (15-18) and (24-26) the results for the pressure, velocity and temperature field for different temperature walls microbearing gas flows and for the low Reynolds numbers are presented in this section. All results are calculated for Λ = 1, σ_v = 1, σ_T = 1, and for a monoatomic gas (κ= 5/3, Pr = 2/3). In the Figures 2-5 the pressure distribution in the microbearing is presented for the slip and continuum regimes. Figure 2 shows that the rarefaction leads to the lower pressure in the microbearing. However, taking into account viscosity and thermal conductivity dependence on temperatures increases the pressure in the microbearing (Fig. 3).

Finally, it is shown in Figure 4 that the wall inclination has the same effect on the rarefied gas flow as well as on the gas flow in continuum regime.

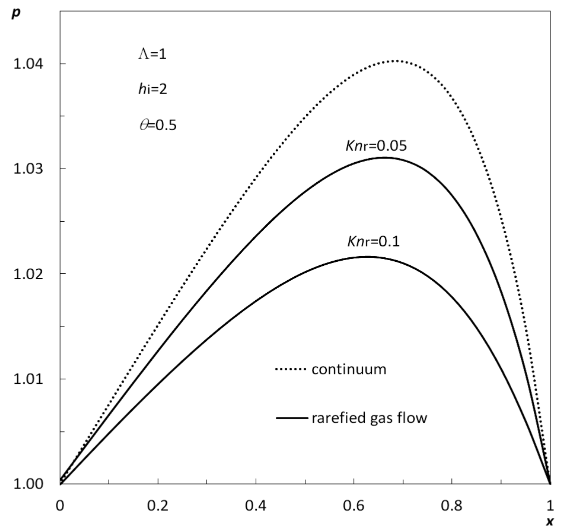

Figure 2. Pressure distribution in microbearing for different Knudsen numbers

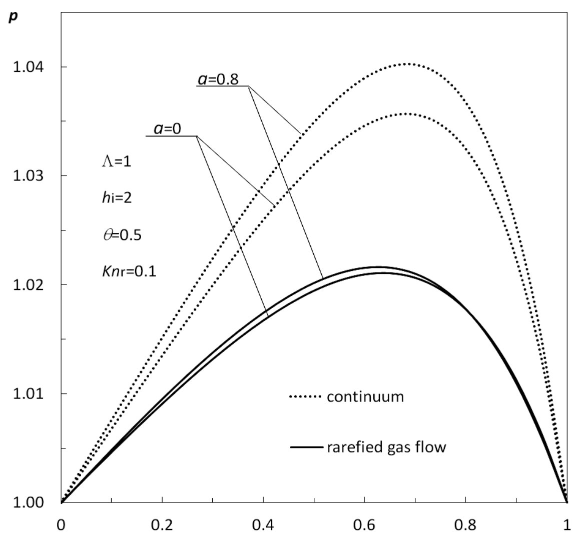

Figure 3. Pressure distribution in microbearing for different parameter *a*

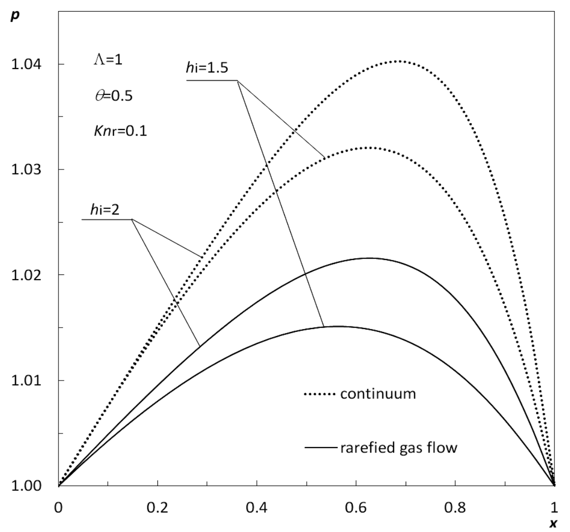

Figure 4. Pressure distribution in microbearing for different slopes

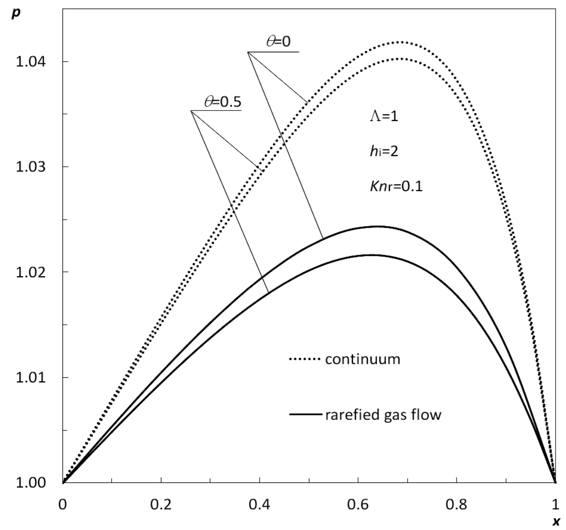

Figure 5. Influence of the temperature of the walls on the pressure distribution in microbearing

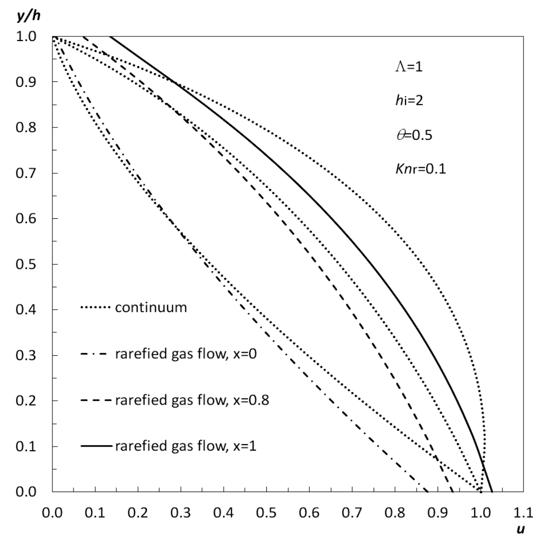

Figure 6. Velocity profiles in the three microbearing cross sections for continuum and slip regimes

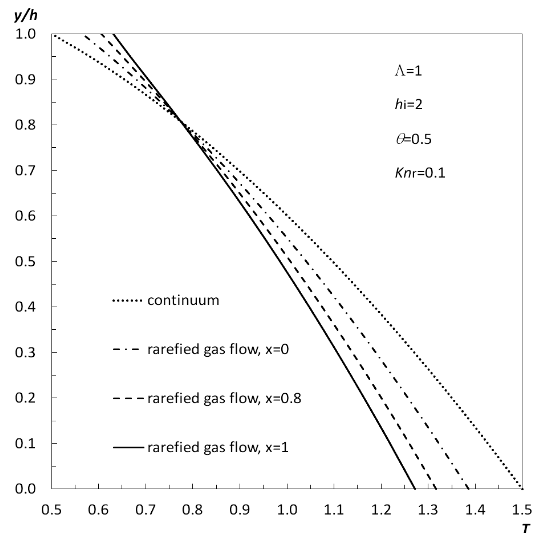

Figure 7. Temperature profiles in the three microbearing cross sections for continuum and slip regimes

Namely, the higher wall inclination corresponds to the higher load caring capacity. Figure 5 illustrates that heating of the lower wall results in pressure decrease in the microbearing.

Velocity profiles in the three microbearing cross sections for continuum and slip regime are presented in Figure 6. It is evident that the slip at the walls is present in all cross sections for rarefied gas flow regime. Also, the temperature jump on the microbearing walls exists in the slip regime (Fig. 7).

4. CONCLUSION

An analytical solution for the non-isothermal subsonic slip microbearing gas flow for the low Reynolds numbers is obtained. The pressure, velocity and temperature fields are found by the perturbation method. The first approximation corresponds to the continuum flow conditions, while the second approximation is a consequence of the rarefaction. It is revealed that the terms in the governing equations which are correspond to the inertia and dissipation can be neglected for the low Reynolds number regime.

The obtained results for slip gas microbearing flow differ from the results for continuum flow conditions. This deviation is evident in the velocity and temperature profiles, as well as in the pressure distribution along microbearing i.e. its load carrying capacity.

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