Serbian Tribology Society

SERBIATRIB '15

14th International Conference on Tribology

University of Belgrade, Faculty of Mechanical Engineering

Belgrade, Serbia, 13 – 15 May 2015

THE INFLUENCE OF THE RUNNING-IN PROCESS OF THE GEAR FLANKS ON THE LOAD DISTRIBUTION IN SIMULTANEOUSLY MESHED TOOTH PAIRS

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Abstract: *Load distribution in simultaneously meshed gear teeth has a very significant effect on the capacity, reliability, energy and environmental efficiency of the gear pairs. Therefore, the choice of shape, size, manufacturing accuracy and materials of the gear pairs should provide an even distribution of workload on the pair of teeth in contact. In order to have a uniform load distribution, it is necessary to establish the appropriate compatibility between the intensity of load, stiffness, manufacturing precision and the amount of the running-in of teeth flanks. This paper examines the impact of the running-in process of teeth flanks on load distribution in simultaneously meshed pairs of teeth, considering different manufacturing accuracy and different levels of load. The analysis was conducted based on analytical expressions for the load distribution factor.*

Keywords: *load distribution, running-in, manufacturing accuracy, cylindrical gears.*

1. INTRODUCTION

Today, there is a growing tendency to pursue optimal mechanical structures in terms of weight, reliability and economy. At the same time, working conditions are becoming stricter with respect to speed, load, heating, vibration and noise. The main question is: how can we find the optimal solution for gear power transmissions, having in mind that they represent vital components of many mechanical structures, with the above-mentioned constraints? New materials, new design solutions for the shape of tooth profile and more accurate calculation methods are proposed. However, the effect of a running-in of gear flanks on the working capacity of gear pair is not sufficiently explained in the scientific and technical literature.

Contact areas of meshed teeth flanks are not ideally smooth. They consist of micro-asperities of different shapes and dimensions and the load is transmitted from one part to another by the tips of those asperities. The actual contact surface is significantly different from the theoretical - ideal contact surface. Surface pressure has a different value at each point of contact. During the work the micro-geometry of the contact surfaces, which are in relative motion, is constantly changing. Surface waviness incurred in the production process is replaced with waviness of different topography. Depending on the tribological conditions in the contact zone, newly formed waviness can be more or less favourable than the incurred one. Each following contact is achieved via the contact surfaces of different geometry, which directly reflects the distribution of load and surface pressure on related parts [1]. The aim of this paper is to discuss the impact of the running-in process of gear flanks on the distribution of load on the flanks of involute teeth profile. The cylindrical gear pairs are considered in this analysis. The analysis was conducted for different materials, geometrical sizes, and manufacturing accuracies of gear pairs.

2. LOAD DISTRIBUTION OF THE MESHING GEAR TEETH PAIRS

Analytical and experimental studies with the aim to determine the actual workload of machine parts, components and assemblies are always current. The results of these studies are the basis for the assessment of their working ability. Primary workload of the mechanical part is the result of its function. Besides the main load, mechanical parts are exposed to additional loads which result from the irregularity in shape, size and relative position of parts, as well as their imbalances. Additional loads mentioned above are particularly present in gear pairs. Therefore, the determination of the actual workload of the gear pair is very complex and economically unjustified in most cases. Despite the theoretical considerations, complex experimental investigations are often needed. Consequently, an analysis of working ability of the mechanical parts and elements is commonly conducted with the assumed loads. Nominal load (F_{nom}) is translated into the assumed one (F_{mer}) using the working conditions factor i.e. the applicable load factor K (Fig. 1).

$$F_{mer} = F_{nom}\,K; \quad K \geq 1$$

The working conditions factor of the gear pair is calculated using the following form:

$$K = K_A \cdot K_V \cdot K_\alpha \cdot K_\beta$$

where: K_A - application factor, K_V - dynamic factor, K_α - transverse load factor, K_β - face load factor.

These influencing factors are analyzed in detail in the scientific and technical literature [1-4]. For the gear teeth working ability, the analysis in terms of volume and surface strength, noise, vibration, efficiency degree and heating requires [5] the knowledge of load distribution on the flanks of the meshed teeth. To ensure continuous transmission of rotary motion from the drive to the driven gear, it is crucial that the next couple starts their own contact before the contact of previously meshed teeth ends. Kinematic indicator of the existence of continuity in transmitting rotary motion is referred to as the contact ratio. According to this criterion, continuous transmission of rotary motion is achieved if the contact ratio is higher than one ($\varepsilon_\alpha > 1$). The contact ratio provides information on the number of simultaneously coupled pairs of teeth which participate in the transfer of load during the contact period:

- $1 < \varepsilon_\alpha < 2$ – one and two pairs of teeth alternately take turns,
- $2 < \varepsilon_\alpha < 3$ – two and three pairs of teeth alternately take turns.

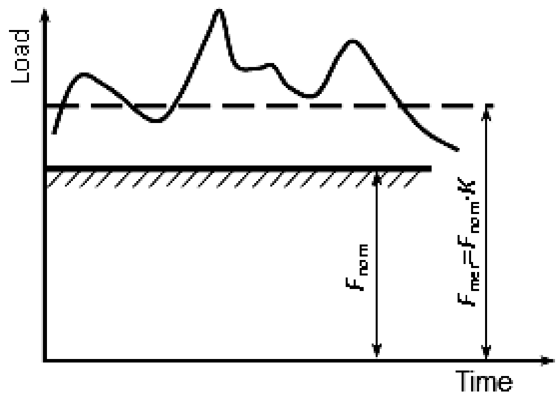

Figure 1. Nominal, assumed and actual load

One of the ways to increase teeth flanks working capacity is to engage a larger number of teeth pairs in the transmission of load. Generally, the load distribution for simultaneously meshed pairs of teeth is uneven. This means that simultaneously coupled pairs of teeth are differently involved in load transmission of the gear pair.

If the total load of gear pair F is simultaneously transmitted over the "n" simultaneously meshed pairs of teeth and if the i-th simultaneously meshed pair of teeth transmits the load denoted with F_i, then the quotient F_i/F shows the engagement level of the i-th simultaneously coupled pair of teeth in the transmission of the total load. Using this force quotient, the load distribution factor of simultaneously meshed pair of teeth can be defined [2]:

$$K_{\alpha\text{-}i} \overset{\text{def}}{=} \frac{F_i}{F}$$

where: $F = \sum_{i=1}^{n} F_i$ – total load of the gear pair, F_i – partial load transmitted by the i-th simultaneously meshed pair of teeth, n – number of simultaneously meshed pairs of teeth.

The manufacturing process of gear teeth brings about deviations in shape and dimensions of the profile. Under the influence of load the elastic deformations of teeth, body and wreath of gears, shafts and bearing supports are generated. During the operation, due to the process of wear and running-in, a change in micro- and macro-geometry of tooth flanks is present. Therefore, it is very difficult to provide a high level of uniformity of load distribution for simultaneously coupled pair of teeth and continuity of this distribution in the gear pair lifetime. Depending on the accuracy of manufacturing, running-in conditions, exploitation, structural and kinematic conditions, load distribution in simultaneously coupled pairs of teeth can be real or boundary – even and extremely uneven. The initial moment of contact in unloaded teeth, when there is a difference between the pitch steps ($p_{b1} > p_{b2}$), is shown in Figure 2a.

Due to the differences in basic pitch steps between the drive and driven gears ($p_{b1} > p_{b2}$), establishing a double loop i.e. engagement of two pairs of teeth in the transmission of load is not possible without the impact of load which has to deform a couple of teeth having contact at points B_1D_2. A mechanical model of the meshing teeth pair, when a transverse base pitch deviation is present, is shown in Figure 2b. Under the influence of load, base pitch deviation (which is a consequence of manufacturing process) is annulled by the elastic deformations of the pair of teeth in contact at points B1D2.

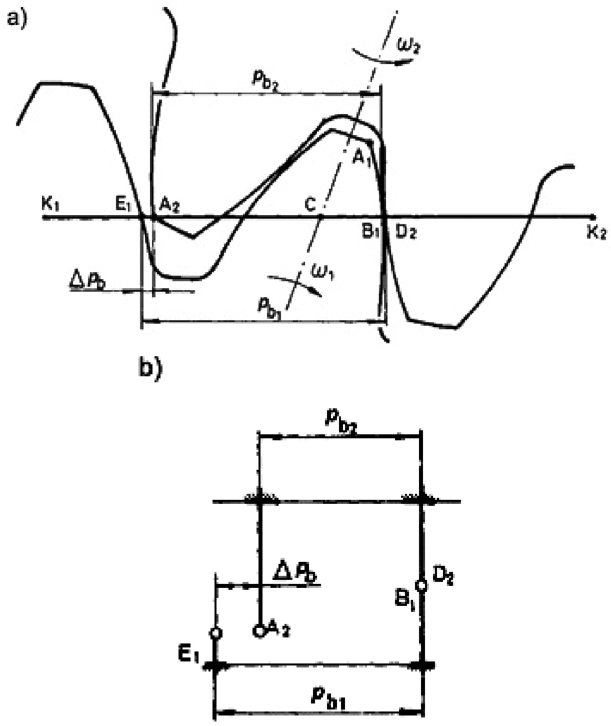

Figure 2. (a) meshing of unloaded teeth during the double loop, when there is a transverse base pitch deviation difference (Δp_b) and (b) mechanical model of meshing

Simultaneously, the second coupled teeth pair is involved in the transmission of load at contact points E_1A_2 (Fig. 2b).

The difference of the base pitch steps has a different value at any point of contact, which depends on the accuracy of manufacturing and running-in of the teeth. During the running-in period, deviation of the profile shape of the teeth is significantly reduced. Accordingly, an actual base pitch deviation is reduced. Reduction of the deviation of profile shape, due to the running-in process, depends primarily on the quality of teeth tolerance (required manufacturing accuracy), stiffness of the meshed pair of teeth, rotational speed, teeth flanks hardness, load and lubrication.

The actual difference between the base pitch steps is replaced with the assumed one:

$$\Delta p_b = |p_{b1} - p_{b2}| = f_{pb} - y_\alpha \qquad (1)$$

where: f_{pb} – base pitch deviation, y_α – running-in allowance.

To describe the distribution of load, a set of simultaneously meshed pairs of teeth is seen as a statically indeterminate system. Based on these assumptions, a functional dependency between the load distribution factor, elastic deformation of teeth, manufacturing accuracy and conditions of the running-in process of teeth flanks has been established [6]:

$$K_{\alpha AE} = \frac{1}{1+\dfrac{c_{BD}}{c_{AE}}} \cdot \left(1+\frac{c_{BD}\cdot \Delta p_b}{F/b}\right) \qquad (2)$$

where: $c_{AE,}$ c_{BD} – the mean value of stiffness of the meshing teeth at contact points AE and BD (wherein $c_{B1D2} \approx c_{B2D1}=c_{BD}$; $c_{A1E2} \approx c_{A2E1}=c_{AE}$), F – determinant tangential load in a transverse plane, b – width of the gear pair, Δp_b – base pitch deviation difference.

Based on the analytical expression (2), the influence of load distribution for simultaneously meshed pair of teeth and the conditions of the running-in process of tooth flanks on the working capacity of cylindrical gear pairs can be analyzed.

3. IMPACT OF THE RUNNING-IN PROCESS ON LOAD DISTRIBUTION

In the initial period of working operation of the newly built mechanical structures all contact surfaces of the coupled machine parts are exposed to the running-in process. During the running-in, mutual engagement of the contact surface (Fig. 3a) roughness peaks occurs. Surface asperities are plastically deformed and/or sheared. Run-in surfaces are smoother and have identical shape (Fig. 3b).

Figure 3. Appearance of the contact areas: (a) before running-in and (b) after running-in

After the run-in period of the contact areas there is a normal working operation period of the mechanical structures.

Wear rate of the contact areas is much lower in this period. The load of gear pairs is transmitted alternately through one or more simultaneously coupled pairs of teeth.

The way in which the total load will be deployed depends on the teeth manufacturing accuracy, the deformational behaviour and the running-in conditions of tooth flanks. The influence of the running-in process on load distribution in simultaneously meshed teeth is taken into account via the running-in allowance. The value y_α is the amount by which the initial base pitch deviation is reduced by running-in from the start of the operation. According to [7] the running-in allowance values are given depending on the type of materials in contact, the tangential velocity of gear pair, the manufacturing accuracy of teeth i.e. base pitch deviation. For various quality grades of the teeth, different surface finish treatments are proposed. One of the indicators of the achieved surface roughness of the gear pair is mean peak-to-valley roughness R_z. Depending on the surface finish of teeth (lapping, polishing, honing, grinding…) the limit values (upper and lower) of R_z are given by ISO [8]. The ratio of running-in allowance and mean peak-to-valley roughness, expressed in function of the surface quality grade and roughness number, is given in the diagram in Figure 4. The area where y_α/R_z < 1 is particularly interesting. In this area, the highest surface peaks, which are the result of the manufacturing process, are annulled without damaging the material surface layers. In this way, by using the running-in process of tooth flanks, the additional surface finishing effect is reached, after which the increase of the real contact areas is achieved [9].

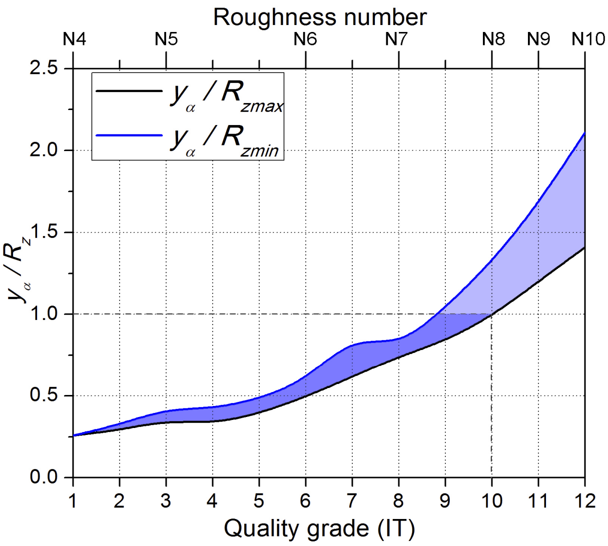

Figure 4. Running-in factor and mean peak-to-valley roughness ratio

Run-in surfaces become smoother. The consequence of these occurrences is increased capacity of the gear pair.

In this paper, we analyze the impact of running-in on load distribution of the gear pair made of normalized low carbon steels.

For these coupled materials, the running-in coefficient is determined by the expression [7]:

$$y_\alpha = \frac{160}{\sigma_{H\lim}} \cdot f_{pb} \tag{3}$$

This expression applies with the following restrictions related to the tangential velocity v:

- for $v \leq 5$ m/s there is no restriction,
- for 5 m/s < $v \leq 10$ m/s the upper limit of y_α is 12 800/σ_{Hlim} corresponding to f_{pb} = 80 µm,
- for $v > 10$ m/s the upper limit of y_α is 6400/σ_{Hlim} corresponding to f_{pb} = 40 µm.

When the materials differ, $y_{\alpha1}$ should be determined for the pinion material and $y_{\alpha2}$ for the driven gear. The average value is used for calculating:

$$y_\alpha = \frac{y_{\alpha1} + y_{\alpha2}}{2} \tag{4}$$

Geometric and kinematic quantities of contemplated gear pairs are given in Table 1.

According to [1,7] the mean value of stiffness of the meshing teeth at contact points AE and BD are:

$$c_{AE} = \frac{4}{11} \cdot c_\gamma \ ; \ c_{BD} = \frac{4}{9} \cdot c_\gamma \tag{5}$$

where: c_γ – mean value of mesh stiffness per unit face width

Transverse single pitch deviation (f_p), caused by the manufacturing process, is given [7] depending on the gear module, reference diameter and quality grade (IT). To determine the load distribution factor according to (2), the base pitch deviation with a higher value is adopted. Substituting (1) and (5) in (2), the load distribution factor is then calculated according to:

$$K_\alpha = 0.45 + \frac{4(f_{pb} - y_\alpha)}{F/b} \tag{6}$$

Based on the expression (6), the analysis was carried out on the impact of the running-in process of tooth flanks on load distribution in simultaneously meshed teeth pairs for different values of the transmission ratio, different materials, and different quality grades (IT6, IT8) of the teeth flanks. The result of the conducted analysis is shown diagrammatically in Figure 5.

Table 1. Geometric and kinematic quantities of contemplated gear pairs

	m_n = 4 mm; $x_1 = x_2 = 0$; $\alpha_n = 20°$				
	Pinion		Driven gear		Transverse contact ratio
Material	DIN EN 1.8550		DIN EN 1.1151		
Allowable stress number σ_{Hlim} [N/mm^2]	770		480		
Transmission ratio	z_1	d_1 [mm]	z_2	d_2 [mm]	ε_α
i = 1	25	100	25	100	1.67
i = 3	25	100	75	300	1.79

Based on diagrams shown in Figure 5, it can be concluded that the impact of the running-in process of tooth flanks on load distribution in simultaneously coupled pairs of teeth increases with decreasing manufacturing accuracy of teeth. With the increase of the intensity of specific load of the gear pair, the impact of the running-in process on load distribution in meshed teeth is reduced. In the area of extremely high specific loads the running-in process has very little impact on load distribution, while in the area of low intensity specific loads the running-in process of teeth flanks has extremely large impact on load distribution.

Different loading modes suit different load distributions. In the areas of extremely low specific loads (F/b = 0 ÷ 300 N/mm), load distribution is extremely uneven. Due to the great effect of the running-in process in this area, the uneven distribution of load can be reduced by more than 30 %. In the field of medium specific loads (F/b = 300 ÷ 600 N/mm), the impact of the running-in on load distribution is reduced and is in the range of

5 – 10 %. With further increase of specific load in simultaneously meshed gear teeth the load distribution factor converges to its ideal value (K_α = 0.5), and the running-in impact on load distribution is decreased. At extremely high specific loads (F/b > 1200 N/mm) the impact of running-in on load distribution is negligible, and tends to a constant value. Reduced impact of the running-in on load distribution in this area does not necessarily mean that this process does not affect the working capacity of the teeth flanks. Teeth working capacity doesn't depend on the load distribution only, but also on the tribological characteristics of the contact surfaces of teeth flanks. In order to consider this statement, detailed experimental studies are needed.

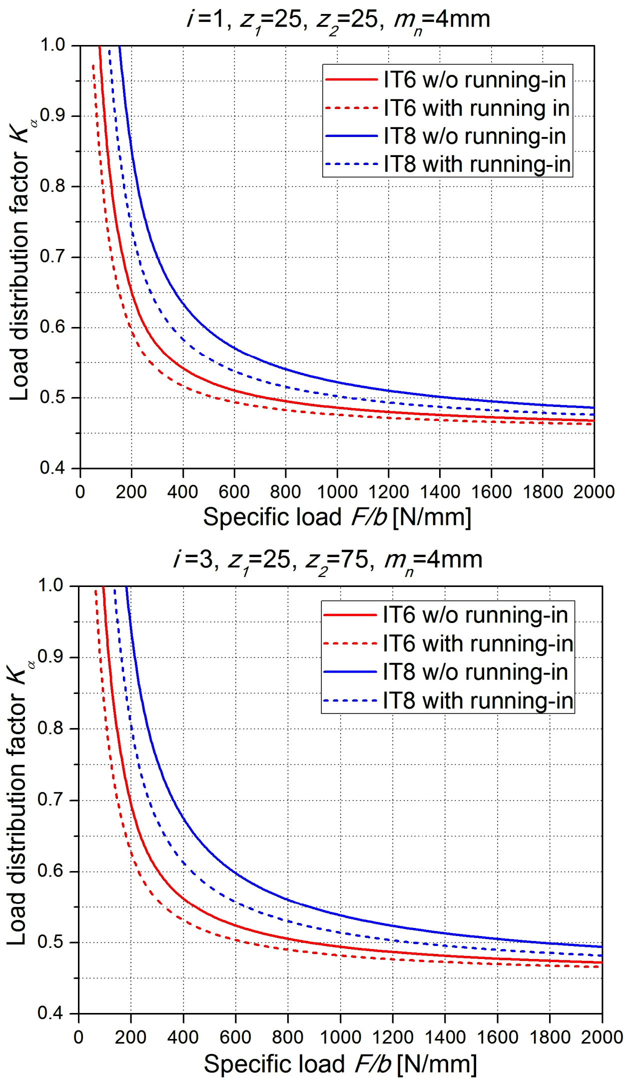

Figure 5. Influence of the running-in process of tooth flanks on load distribution in simultaneously meshed teeth pair

With an increase of the transmission ratio, i.e. the centre distance of the gear pair, the effect of the running-in is increased too. For the gear pairs operating at higher transmission ratios, significant changes of the load distribution factor were observed, compared to the ones operating at lower transmission ratios. This occurrence reflects the fact that base pitch deviation depends on the size of the gear pair. It is noticeable that the intensity of the running-in is inversely proportional to the intensity of specific load.

Also, the effect of running-in depends on the type of material of meshed gears. The teeth made of lower- quality materials (grey or nodular cast iron) have a higher degree of sensitivity to the running-in process compared to the teeth made of high-quality materials (case-hardened steel, flame or induction hardened steel…). The conclusion mentioned above was adopted on the basis of the diagram shown in Figure 6. This occurrence can be explained by the fact that the removal of the tops of the highest asperities under the effect of load on the flanks of teeth is more intense with the materials of lower quality due to lower surface strength.

Also, high-quality materials are commonly treated by the finishing process of mechanical or thermo-mechanical treatment after the manufacturing process in order to achieve greater accuracy and higher strength of the teeth flanks.

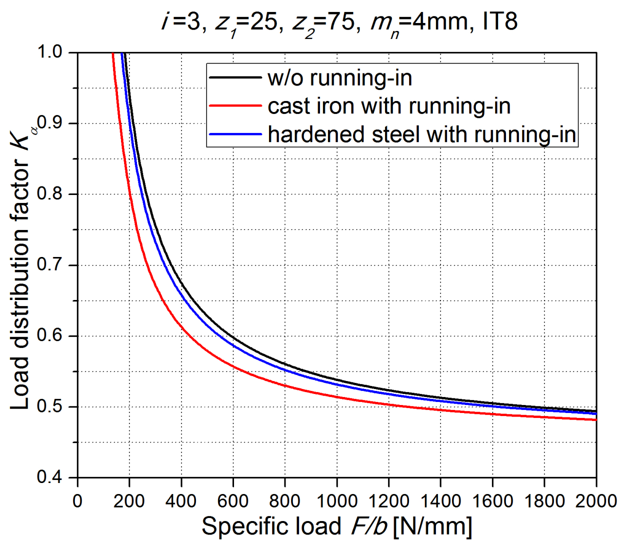

Figure 6. Sensitivity of low-quality cast iron and high-quality case hardened steel to the running-in process

4. CONCLUSION

The analysis conducted in this paper has shown that the running-in process of teeth flanks affects the distribution of load in simultaneously meshed pairs of teeth. This effect is more pronounced in the areas with relatively low specific loads. Based on the results obtained from conducted analysis, the following conclusions are made:

- the impact of the running-in process of teeth flanks on load distribution in simultaneously meshed teeth pairs increases with decreasing manufacturing accuracy,
- with increasing centre distance of the gear pair, the effect of the running-in process on load distribution is also increasing,
- gear teeth made of lower quality materials have a higher degree of sensitivity to the running-in process compared to gear teeth made of higher-quality materials, and
- unevenness of the load distribution can be reduced up to 30 %, using the running-in process of teeth flanks.

Experimental studies are a logical continuation of this work to confirm the above mentioned conclusions.

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