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## MODELLING OF NONLINEAR DYNAMIC OF MECHANIC SYSTEMS WITH THE FORCE TRIBOLOGICAL INTERACTION

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**Abstract:** *This paper considers the mechanisms with different structure: tribometric device and a mechanism for handling of optical glasses. In the first device, the movement of the upper platform is due to a reciprocating friction interaction. In the second device, the processing of the optical element or group of elements occurs due to the rotational motion. Modelling of the dynamic of these systems with Matlab/Simmechanic allowed carrying out the analysis of dynamic of mechanisms considering nonlinearity tribological interactions for these systems. The article shows that using the computer models can effectively carry out the selection of the control parameters to create the desired mode of operation, as well as to investigate the behaviour of systems with nonlinear parameters and processes of self-oscillations. The organization of the managed self-oscillation process is realized to create the relevant high-performance manufacturing, for example, for the processing of optical glasses.*

**Keywords:** *handling of optical glasses, tribometric device, friction models, nonlinearity tribological interaction, simulation in Matlab.*

### 1. INTRODUCTION

In present time, one of the most important problems is the research of the frictional systems, what are based on the interaction the surfaces of different configurations with rough plane or a surface of complex shape. The results of such research can positively influence to the development of braking or grinding tools and machinery in general. Feature of such systems is the presence of intermediate liquid and viscous lubricants and emulsions, complicating their theoretical research and prognostics results. Today in the optics manufacture and other high-tech industries use different types of machines: grinding, polishing, lapping, etc. All these machines are united by the same principle of the working

bodies, based on the reciprocal shaping and torque transferring from the processed tools to the body using the frictional forces. This fact impose some difficulties for machine researching and modernization, these include: the development of non-contact methods of control of the surface state, the identification of self-oscillatory processes and their impact on the establishment of processing modes, accounting elastic and thermoelastic deformations, analysis tool path; construction of feedback algorithms, creating of mathematical models for the dynamic processes prediction, creating of mathematical models for the magnitude of material removal prediction in the treatment zone.

For the identification of friction and self-oscillation research was developed imitating

model of the tribometric device “Tribal”. This device was created at the Department of Mechatronics in ITMO and patented [1]. The kinematic diagram of the device is shown in Figure 2. Matlab simulation model of the “Tribal” is shown in Figure 3a. These models are obtained for different operating conditions, were compared with experimental values.

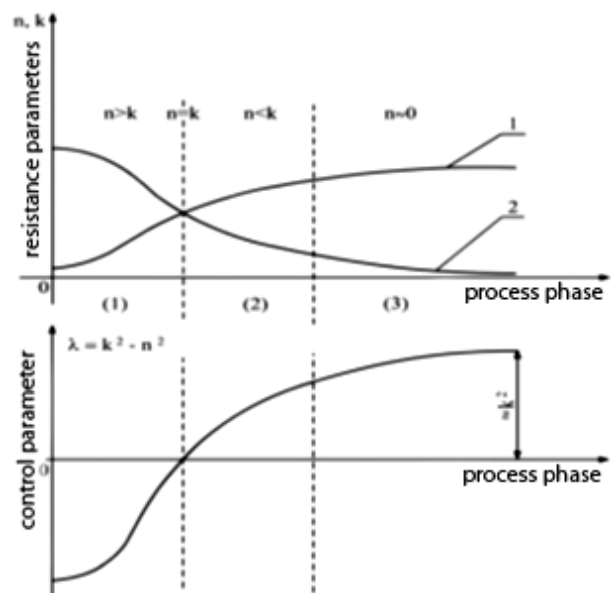
For studying the dynamic in grinding and polishing devices for processing optical glass an actuator imitating model SimMechanics was created, Fig. 4a. The main objective of this study is to investigate impact on the process fluctuations in friction, to define the dynamics of machine for grinding and polishing operations by free lapping and studying level changes of surface roughness using a non-linear friction law.

## 2. THEORETICAL JUSTIFICATION

At the relative motion of solid bodies carried dynamic interaction roughness of the contacting surfaces. It is occurs the deformation zone of reciprocal contact in conjunction with the molecular interaction processes and the process of heat deterioration. According to the theory of Bowden and other [2], friction force is mainly due to two reasons: on the one hand, by cutting adhesion links formed in the areas of actual contact of solid bodies, and the other – ploughing the less rigid surface material of the interacting bodies [3,4]. Mediums, located between the contacting surfaces, have a decisive influence on the friction between the roughness and the physic-chemical processes in the environment. For the automatic control and predicting wear processes of interest are various models of friction. At present, there many physical, mathematical, and analogue simulation models, which describe the phenomena occurring in the contact surfaces with different types of friction [5,6]. Friction is interpreted as a physical interface between the contacting surfaces. Static friction models include: models of dry and viscous friction on the basis of Coulomb's law, Karnopp model, and Armstrong model for describing the

dynamic processes in the classical model of friction. To describe the process of friction also use various nonlinear models using the power, quadratic resistance, as well as methods of catastrophe theory.

In the work [1], in the study of dynamics tribological interaction using the device “Tribal” it was found that the surface damage begins when the dynamic system passing the bifurcation point. During the tests, conducted systematically, assessing dynamic models, corresponding to the process of friction. At each stage the system identified two characteristics: the impulse response and the identity a transitional function. Thus, in sliding friction registered second order phase transition, or Hopf bifurcation, after which begins the process of deterioration. Calculated scale of the phenomenon, which allows consider the emergence of an additional degree of freedom (Fig. 1). Based on these results, it was decided to use the theory of catastrophes, for studying the problem of friction for the processing of optical glasses.

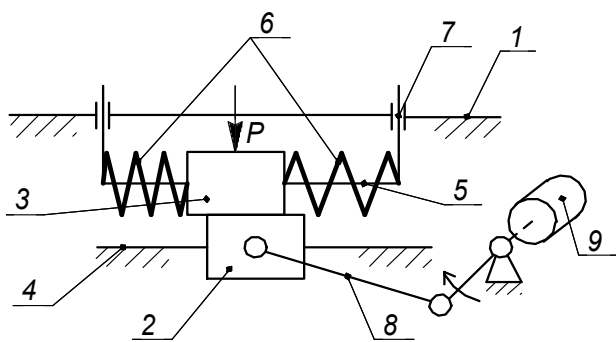


**Figure 1.** Changing the damping coefficients and natural frequencies:  $n$  – damping factor,  $k$  – natural vibration frequency,  $\lambda = k^2 - n^2$  – natural parameter

## 3. MODELING THE DYNAMICS OF THE TRIBOMETRIC DEVICE “TRIBAL”

Tribometric device (Fig. 2) consists of a base 1, two platforms: the lower 2, and upper 3.

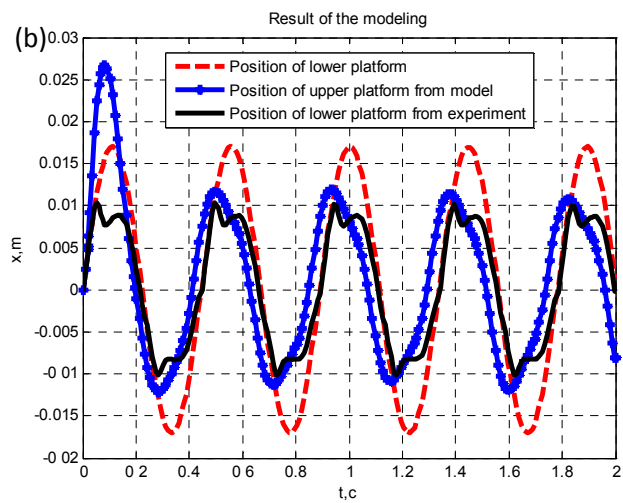
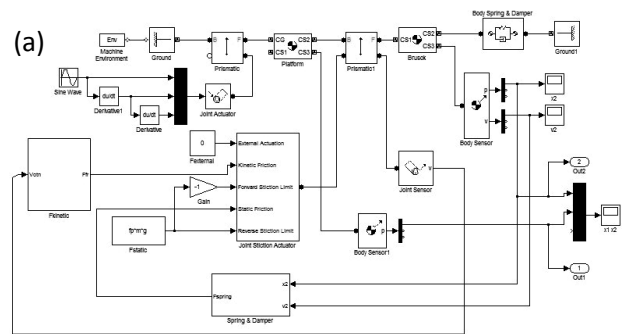
Platforms are capable of reciprocating movement along the rails 4 and 5, respectively. The upper platform can move in the vertical direction due to the guide 7. On the upper platform acting forces by the springs 6, mounted on the rail 5, as well as vertical force (P). Lower platform is driven by the motor 9 via a crank mechanism 8 and transmits movement to the upper platform due to frictional forces. Under the action of the springs, the upper platform tends to return to its original position and thus oscillates. Platforms position are determined by linear encoders.



**Figure 2.** Kinematic scheme of the tribometric device: 1 – base; 2 – lower platform; 3 – upper platform; 4 – lower platform rail; 5 – upper platform rail; 6 – springs; 7 – vertical guide; 8 – crank mechanism; 9 – engine

In Figure 3a is shown the animated SimMechanics model of tribometric device built by using a blocks Matlab/Simulink/Simmechanics. Vertical force in the model is taken constant and given the mass of the upper platform. Spring and damper block simulates the elastic-dissipative forces depending on the set of coefficients of elasticity and damping. Friction force depend of the speed of the platforms relative motion, and given by block Joint Stiction Actuator (Drive unit with “sticking”). The results of plate movement define the group of sensors blocks. More details, the model of friction is considered in [4]. To the model build are used the equations for frictional relaxation self-oscillations given from [4,7]. For the modelling a system are used standard blocks of package Simulink/Simmechanics and considered such factors as the difference in the coefficients of friction for the rest and sliding, dependence of

the friction coefficient on the relative velocity, the coefficients of elasticity and the damping of the elastic system. The results of calculation and simulation are shown in Figure 3b.



**Figure 3.** (a) animated model SimMechanics and (b) comparative graph of experimental data and model data

Resulting graphs show the performance of the tribometric system. Under the action of friction, the upper platform, lying on the lower, is involved in a joint motion. The spring, fixed to the upper platform, when the resilient force exceeds the force of static friction, shifts upper platform relative to the lower. The graphs show that the simulation model data correlate well with the experimental ones. The degree of convergence of the results is of approximately 85 %. The results of the friction model in the presence of frictional self-oscillations are identical with the analytical solution, given in [8].

The considered model of friction can be used to study the dynamics of various mechanical systems, where it is important to consider the effect of friction. Research of dynamic processes such systems allow to make adjustments to their work and choose

the most effective parameters to improve the efficiency of their use.

#### 4. MODELING THE WORKING ASSEMBLY OF GRINDING WITH NONLINEAR FRICTION

Current level of development of optical production requires continuous upgrading of equipment, the introduction of automation with the use of computer technology, research processes of shaping surfaces. It is necessary to control the following parameters: the deviation of surface shape from the desired; deviations mutual disposition polishing surfaces relative to baseline, the change of surface roughness, the quality of the image constructed optical surfaces [3,4]. These issues at different times were engaged scientists Tsesnek LS, VA Smirnov, A. Bardin and others. Exception – machines, the principle of which is based on forced shaping details. Almost all of the above problems require a coherent analysis of the dynamics and kinematics in general. The relevance of this problem is mentioned in [3-9]. Using the experience of research described in the second section, created a simulation model of the actuator for processing of optical elements (Fig. 5).

Figure 4a shows a functional diagram of the mechanism for the polishing optical glasses. Scheme is conditional and includes: gear and the spindle motor  $D_1$  on one side, and crank mechanism for the leashes swing and motor  $D_2$  on the other. There are lower faceplate 1 with fixed on it the workpiece 2, the torque transmitted from the engine  $D_1$  to lower faceplate. Engine  $D_2$  transmits torque to the joints 5 and 6, which provide the necessary law of motion of the upper faceplate (polishing). It should be noted, that to prevent the discontinuity of the kinematic connections between the elements 3 and 5, as well as 2 and 3, is introduced into the scheme power P. In this case there is a force connection between the bodies operating elements of the kinematic scheme. The entire mechanism is immersed in the emulsion tank with circulating 4, which is continuously supplied to the contact surface. It cleans them from wear

products and polishing.

Dynamic processes occurring in any mechanisms, depend essentially on the properties of its mechanical parts. Therefore it is necessary to analyze the movable joints of the kinematic chain in mechanisms. Thus, the shaft 2 transmits rotary motion from the rack  $O_2$  (Fig. 4b) and forms a fifth-grade kinematic pair with a conditional rack  $O_2$ , forcing mechanism specified rotation around axis. The shaft with a crank forms a fifth-grade kinematic pair and allows the machine to rotate around axis. Crank and polishing pad form a third kinematic pair [7]. Third-class couple B allows the machine to perform the rotation around axis. Due to the hinge B, the inner surface of the tool is mounted itself on the workpiece surface. Polishing pad and the workpiece forms a third-class kinematic pair A, and rotation around axes: The workpiece is rigidly fixed to the lower face plate which is immovably fixed to the shaft 1. The shaft transmits the rotary motion of the rack  $O_1$ , and forms one fifth-grade kinematic pair with a conditional rack, reporting mechanism specified rotation around the axe. In the A and B kinematic pairs is provided the constant contact by the force bridging.

Character movement of engine  $D_2$  is considered as reciprocating-rotational about an axis. Considering the kinematic scheme (Fig. 4b), we assume that the input links are shafts 1 and 2, and the output link – polishing tools. The mechanism has three degrees of freedom. Consequently, to determine the movement of all links should be given three generalized coordinates. Make sure to consider that the mechanism for the polishing of optical glasses relates to mechanisms with a variable mass units. This is due to the mutual wear of a workpiece and a pad. At research the mechanism is necessary to lay the law to wear. Mechanical part is conveniently considered as a system with holonomic, stationary and ideal retaining constraints. Also applied to the system of the generalized driving forces, acting on the links of the mechanism and the resistance forces, applied to the links of the actuators [7,10].

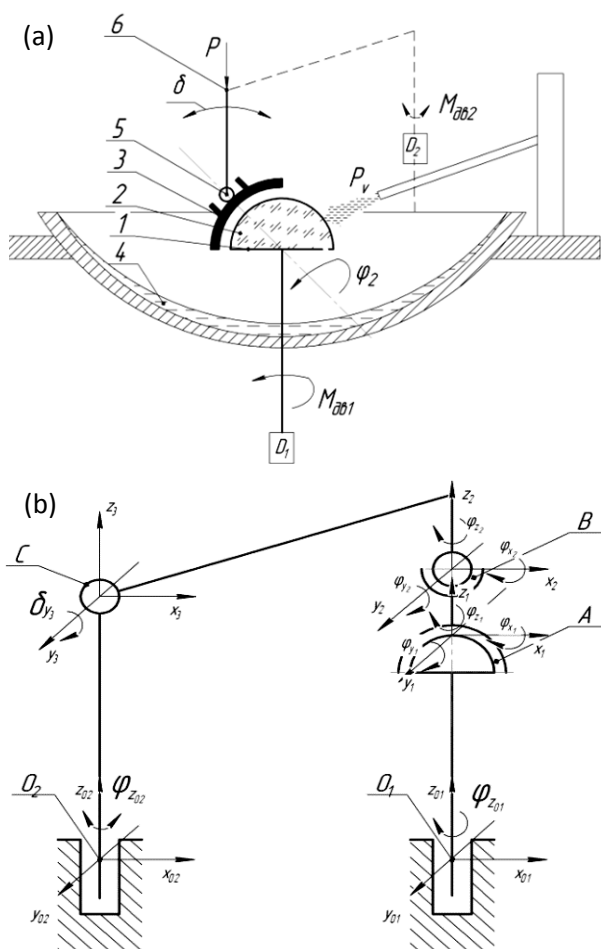
Suppose that all the kinematic pairs of the mechanism, except pairs A and B, are ideal, and all links rigid, then:

$$L_s\{q_1, \dots, q_n, Q_1, \dots, Q_j\}. \quad (1)$$

In this case, to describe the dynamics of the mechanism is sufficient to introduce three generalized coordinates:

$$\begin{cases} \varphi_{z_{01}} = \text{const} \\ \varphi_{z_{02}} = \text{const} \\ \delta_{v_3} = f(t) \end{cases} \begin{cases} \varphi_{x_1} = \varphi_{x_2} = q_1 \\ \varphi_{y_1} = \varphi_{y_2} = q_2 \\ \varphi_{z_2} = q_3 \end{cases}, \quad (2)$$

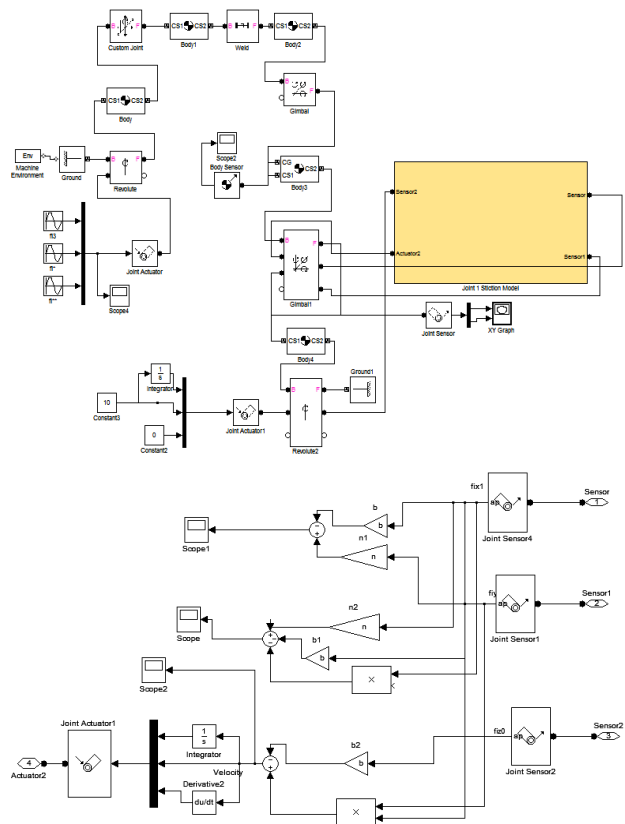
where:  $z_{01}$ ,  $\varphi_{z_{01}}$  and  $z_{02}$ ,  $\varphi_{z_{02}}$  – constant as engine performance is considered to be ideal.



**Figure 4.** (a) functional diagram of an optical glass polishing assembly and (b) kinematic diagram of a polishing optical glasses assembly

Figure 5 shows animated Matlab/Simulink/SimMechanics model of working mechanism for processing of optical elements by free lapping. In the simulation used nonlinear friction law proposed in [1]. It is believed that c and b parameters related to the

characteristics of the intermediate layer. It influence affects for the performance of the working mechanism for the optical element polishing.

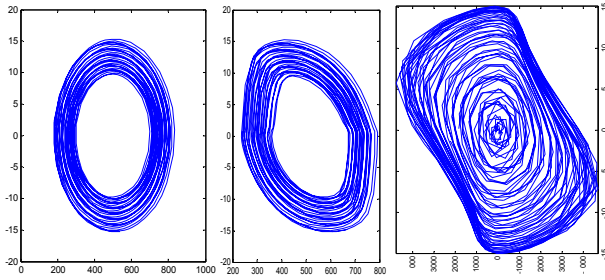


**Figure 5.** Animated model SimMechanics of the mechanism for the optical element polishing

The comparison of the tools phase trajectory made using the data, obtained by the theoretical methods and from the model. The comparison showed that the system with linear friction law does not coincide with the analytical findings. With the gradual introduction of nonlinear coefficients, since the Kelvin-Voigt model, the results improved. In the process of modelling the dynamics the system behaviour changed from steady state (where the phase space subsystems constituted the center) to unstable (where the phase space subsystems has adapted to the steady focus, and then in the unstable focus) and system and acquired a damped oscillatory.

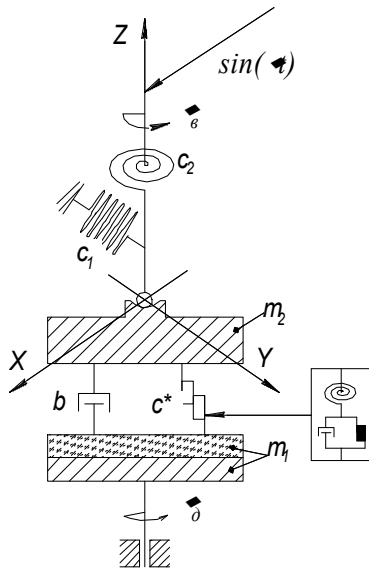
As the extent that the torque M applied to the lower faceplate, the system becomes unstable in all cases. This phenomenon takes place when there are equality of magnitude component damping and stiffness. It follows, that use the extremely viscous fluids for the

polishing leads the irrational use the driving forces of the system. Form of the all phase trajectories of the third body (Fig. 6) indicates the presence of self-oscillating process.



**Figure 6.** The results of the animated model SimMechanics of the mechanism for the optical element polishing

Next model actuator spindle block grinding machine in processing of flat optical elements were synthesized (Fig. 7).



**Figure 7.** The equivalent circuit of the actuator for flat optical elements

The lower chuck aligned with the optical element, a mass  $m_1$ , which fixed on it. At the bottom faceplate acts input torque from the motor  $M_1$ . The angular velocity of the upper faceplate mass  $m_2$  transmitted, through elastic-dissipative layer. It is characterized by two components: the dissipative body, which has a viscosity coefficient  $b$ , and visco-plastic body  $c^*$ , whose properties are expressed by equation 3:

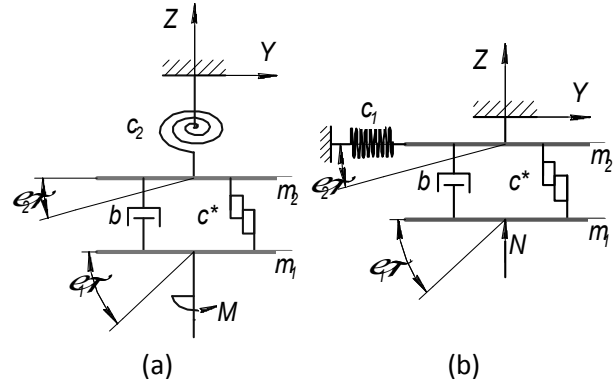
$$q_n + \mu \cdot \dot{q}_n = c^* \quad (3)$$

where:  $\mu$  – plastic viscosity,  $q_n$  – generalized coordinate.

The spindle elasticity pronounced by stiffness coefficient of the elastic element  $c_2$ , which allows the system to work on twisting. The stiffness coefficient of the elastic element  $c_1$  provides the system ability to move under the influence of  $P(t)$  of the motor  $M_2$ . This effect is permanent, alternating sign and can be expressed by the equation 4:

$$P(t)_{\text{pov}} = A \cdot \sin(\omega t) . \quad (4)$$

The equations of motion a pair of tool-workpiece can be found by expanding the system into two equivalent schemes (Fig. 8), corresponding to an independent degrees of freedom: rotation about the Z axis and displacement along the plane XY. The directions of these DOF are shown in Figure 7. Elastic-dissipative layer, that located between the bodies, working in all directions of motion pair tool-workpiece: the twisting and moving, so we can assume, that the intermediate layer is isotropic. Equivalent scheme (Fig. 8) is described by a system of two equations with three additional (Formula 5):



**Figure 8.** The equivalent schemes for the calculation: (a) scheme of the executive system axis on the Z-axis and (b) scheme of the executive system axis on the Y-axis

$$\begin{cases} J_2 \ddot{\varphi}_2 + (\varphi_1 + \mu \cdot \dot{\varphi}_2)(\dot{\varphi}_2 - \dot{\varphi}_1) + c_2 \varphi_2 + \\ \quad + b(\dot{\varphi}_2 - \dot{\varphi}_1) = M_{\text{dv}}(t) \\ m_2 \ddot{y}_2 + (y_1 + \mu \cdot \dot{y}_2)(\dot{y}_2 - \dot{y}_1) + c_1 y_2 + \\ \quad + b(\dot{y}_2 - \dot{y}_1) = P(t)_{\text{dv}} \end{cases} \quad (5)$$

where  $\varphi_1$  – the initial angle of the bottom faceplate, its position is known,  $\varphi_1 = 0, t = 0$ .

$$\begin{cases} (\varphi_1 + \mu \cdot \dot{\varphi}_2)(\dot{\varphi}_2 - \dot{\varphi}_1) + b(\dot{\varphi}_2 - \dot{\varphi}_1) = N \\ (y_1 + \mu \cdot \dot{y}_2)(\dot{y}_2 - \dot{y}_1) + b(\dot{y}_2 - \dot{y}_1) = F_{\text{tr}} \end{cases} \quad (6)$$

Thus, in the systems of equations 5 – 6 there are five unknowns:  $\varphi_2$ ,  $y_2$ ,  $y_1$ ,  $F_{tr}$ ,  $N$ . It should be noted, that the absence of contact the upper faceplate with workpiece by elastically-dissipative layer leads to absence of mutual abrasion surfaces. The contact surface is the main kinematic condition. In case of breakage contact the inertial motion emulsion layer consideration should also be neglected due to its small thickness and speed. Coordinates X and Y are interrelated expression 7:

$$x^2 + y^2 = L^2. \quad (7)$$

where:  $L$  – length of the carrier.

Elastic-dissipative characteristics of the system are calculated from the theoretical calculations. In addition, it is assumed that the evolution of the system parameters occurs considering the three main stages, presented in Figure 6.

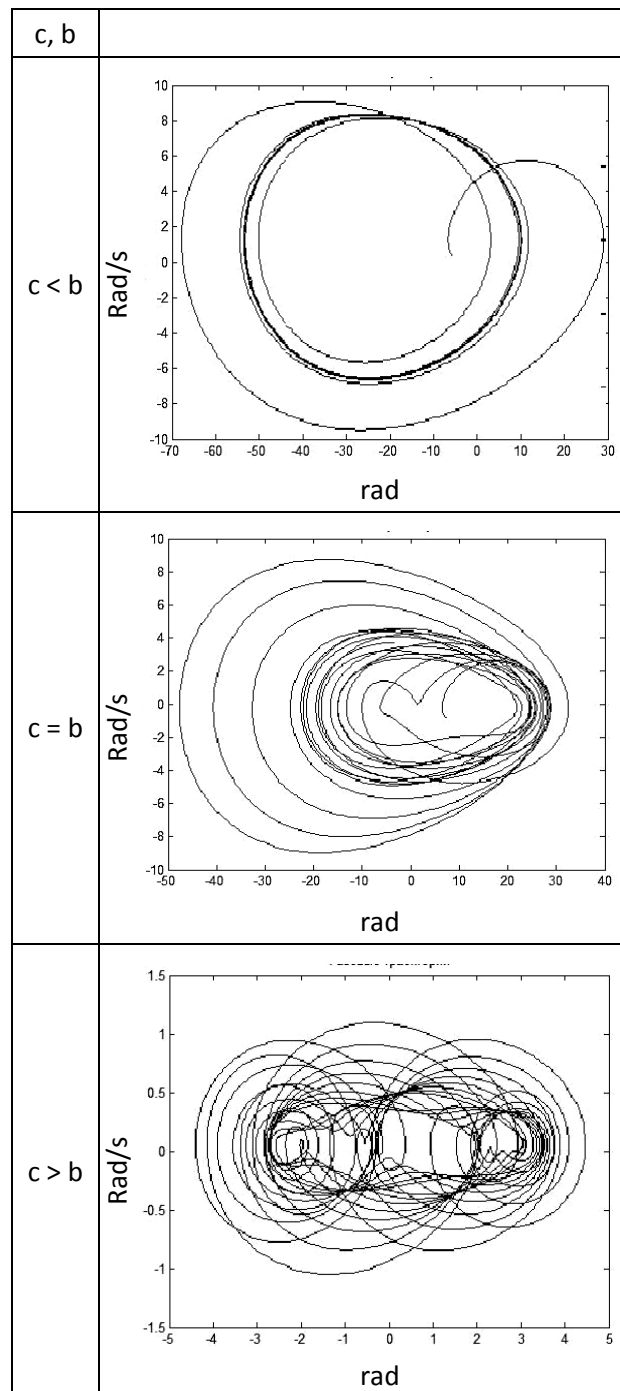
Initial parameters are selected on the basis of a flat lens processing mode, considered in [1]. The modelling results obtained by the equation are presented in Table 1.

## 5. CONCLUSION

On the basis of experimental studies using tribometric device “Tribal” was developed animated simulation model SimMechanics. This allowed to compare the experimental and theoretical results, as well as to establish their degree of convergence. After the positive results animated model SimMechanics was created of the main mechanism for the processing of optical elements. The necessity of this approach due to the fact that the technological features of the machines do not allow us to study the internal processes experimentally. Several models have been created for different configurations of machine and compared [10-12]. Carried out theoretical calculations, and picked up the laws of friction for various stages of processing. Research the friction models is going using two systems: the model “Tribal” and the model of machine for the processing of optical elements. In the future, it is planned

to make the selection of the control parameters to determine the optimal properties of the polishing suspension in relation to high-speed mode of the machine.

**Table 1.** The modelling results



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