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SOME ASPECTS CONCERNING THE BEHAVIOUR OF FRICTION MATERIALS AT LOW AND VERY LOW SLIDING SPEEDS

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Abstract: *The tribological aspects concerning the behaviour of friction materials in the range of low and very low sliding speeds (0.2...200 mm/min) are essentially different of those in the high sliding speeds range. This paper aims to study the stick-slip phenomenon which occurs in the range of low and very low speeds. Typically, for the stick-slip phenomenon to occur, the static friction coefficient between the two contact surfaces of the friction materials must be larger than the kinetic friction coefficient. For disc brakes, the stick-slip phenomenon is mentioned in many specialized scientific papers. The phenomenon is manifested through self-induced vibrations. The experimental results were obtained on a dedicated testing machine where the parasite stick-slip motion is reduced through the use of bearings.*

Keywords: *sliding speed, disc brake, stick-slip, automotive brake friction materials, self-induced vibrations, friction coefficient.*

1. INTRODUCTION

In the tribological domain there are still many fundamental problems that have not yet been completely elucidated because of the complexity of the phenomena. Of these, we mention: the relations between the static and kinetic friction coefficients, the static friction coefficient's dependence of idle time and the kinetic friction coefficient's dependence of the speed and acceleration of the movement etc. Such problem is the stick-slip phenomenon [1].

At low sliding speeds, in dry, limit or mixed friction conditions, the movement can have intermittences or jerks. This phenomenon is called stick-slip. Typically, for the stick-slip phenomenon to occur, the static friction coefficient (μ_s) between the two contact surfaces of the friction materials must be greater than the kinetic friction coefficient (μ_k).

As it is known, the stick-slip phenomenon appears in friction couples with dry or limited friction regime, when the sliding speed is in the range of 0.01-3 mm/s or when the angular speed is somewhere in the 1-25 rad/s [2,3]. If these speeds have higher values, then the movement takes the form of self-induced vibrations sustained by the friction force itself [4].

In the general case, μ_s and μ_k can be complicated functions of sticking time and surface speed, respectively. Furthermore, static friction is a constraining force during sticking, while kinetic friction is an applied force during slip [5].

In their paper, Gao et al. [5] derived a theoretical equation in which the growth rate of the static friction force is influenced by the system damping and the speed dependence of kinetic friction. Starting with a general dynamic analysis, they showed that $\mu_s > \mu_k$ is a necessary but not sufficient condition for tick-slip motion, that $d\mu_s/dt$ is a crucial parameter and that stick-slip can occur even if μ_k increases with speed. Explicit equations based on a general $\mu_s(t)$ and a linearized $\mu_k(v)$ were developed for determining the slip mode, the stick-slip amplitude, the critical substrate speed above which stick-slip ceases, and the saturation substrate speed below which the stick-slip amplitude is constant. They also made comparisons between the theoretical predictions of those equations and a wide range of experimental observations on a hoop apparatus [5].

The stick-slip phenomenon is characterised by vibrations caused by friction interactions between sliding surfaces. It is the cause of all types of brake noise: squeal and squeak (high frequency: ≈ 0.6 – 2 kHz), and moan, groan, judder and chatter (low frequency: < 0.6 kHz). The paper aims to develop a theoretical model through which we can study the behaviour of the brake friction couple materials at low speeds, when stick-slip occurs.

Another objective of the paper is to highlight the critical force and implicitly the critical contact pressure at which the stick-slip phenomenon occurs at the specific contact of the friction materials for the automotive disc brake system. The critical contact pressure can be determined by relating the critical force to the surface of the slider, which in our case represents the brake pad. The contact surface of the slider has an area of 105.68 mm, so the critical pressure will be $p_{cr} = P_{cr}/105.68$ N/mm, where P_{cr} is the critical force.

In this paper we adapted Gao et al. [5] and Bo and Pavelescu's [6] equations so that they could be used for the study of the experimental results obtained with the testing apparatus from our department specially designed for studying friction and wear at low and very low sliding speeds. The results of the theoretical model are graphically analysed for the seven different sliding modes described in the paper.

2. THE TESTING APPARATUS FOR LOW AND VERY LOW SLIDING SPEEDS

The testing apparatus from The Department of Machine Elements and Tribology, University "Politehnica" Bucharest, used for the current experiments is specially designed for the study of friction and wear of materials at low and very low speeds, the study of thermal stability of oil additives, and the study of the stick-slip movement. Figure 1 presents a general view of the apparatus. In the experiments we conducted, the moving sample (slider), made out of brake pad friction material, slides back and forth across the fixed sample, made out of brake disc rotor gray cast iron.

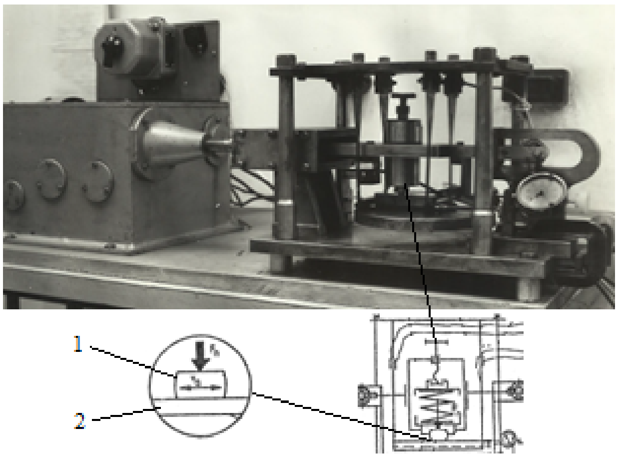

Figure 1. The testing apparatus (1 – slider, 2 – fixed sample)

The driving system is equipped with a DC electrical motor with variable speed and a mechanical gear box. Such a system allows a variation of the speed through the whole speed range 0-50 mm/min. To eliminate the parasite vibrations, the driving system is isolated from the support of the apparatus by bearings.

In Figure 2 we have a stick-slip period caught in a series of initial experiments we have done with the brake pad – brake disc rotor samples. Future experimental work must be done to validate the theoretical model elaborated in this article.

3. THE THEORETICAL MODEL

The theoretical model developed herein consists of an adaptation of Gao et al. [5] and Bo and Pavelescu's [6] equations so that they could be used for the study of the experimental results obtained with the testing apparatus from our department specially designed for studying friction and wear at low and very low sliding speeds.

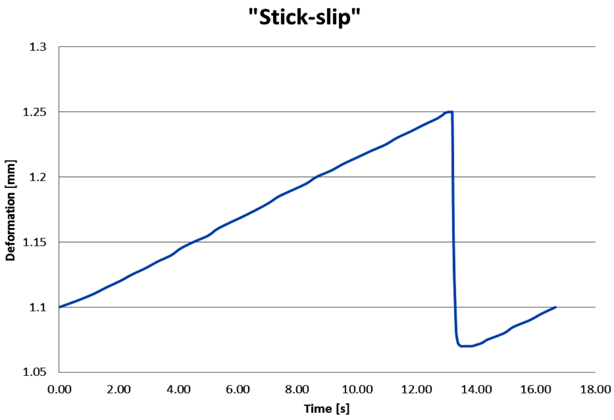

Figure 2. Stick-slip period

It is assumed that kinetic friction coefficient μ_k is dependent on the speed [5]:

$$\mu_k = \mu_0 + \alpha v_r, \tag{1}$$

where μ_o and α (the coefficient of the linear term for μ-V curve) are arbitrary constants and

$$v_r = v_0 - \frac{dx}{dt} \tag{2}$$

is the relative speed between the slider and the fixed sample. Parameters μ_o and α will be determined by experimental method with our testing apparatus (Fig. 1). The kinetic friction coefficient for the sliding movement without the stick-slip phenomenon is:

$$\mu_{k0} = \mu_0 + \alpha v_0, \tag{3}$$

where v_0 is the velocity of the slider.

The effective damping coefficient β of the measuring system is considered to be:

$$\beta = \frac{\gamma + \alpha P}{2 m_s}, \tag{4}$$

where γ is the damping coefficient of the measuring system, P is the normal force and m_s is the mass of the fixed sample.

The oscillation frequency in harmonic motion is:

$$\omega = \sqrt{\frac{k}{m_s}}, \tag{5}$$

where k is the spring constant of the measuring system.

For the case in which the harmonic motion is damped ($\beta > 0$) or pumped ($\beta < 0$), the oscillation frequency will be:

$$\Omega = \sqrt{\omega^2 - \beta^2} \tag{6}$$

The differential equation of motion during slip phase will be:

$$\frac{d^2x}{dt^2} + \frac{\gamma + \alpha P}{m_s}\frac{dx}{dt} + \omega^2 (x - x_0) = 0, \tag{7}$$

where

$$x_0 = \frac{P}{k}\mu_{k0} \tag{8}$$

is the time-averaged position of the slider.

The solution for the motion equation (7) is:

$$x(t) - x_0 = A e^{\lambda_p t} + B e^{\lambda_m t}, \quad \text{if} \quad |\beta| \neq \omega \tag{9}$$

$$x(t) - x_0 = A e^{\beta t} + B t e^{\beta t}, \quad \text{if} \quad |\beta| = \omega \tag{9'}$$

where A and B are integral constants, determined from the initial conditions, while

$$\lambda_p = -\beta + i(\omega^2 - \beta^2)^{\frac{1}{2}}, \tag{10}$$

$$\lambda_m = -\beta - i(\omega^2 - \beta^2)^{\frac{1}{2}}, \tag{10'}$$

are two roots for the second-order auxiliary equation.

Based on the values of the effective damping coefficient β, which can be positive, negative or zero, there can be seven different sliding modes. In these cases sliding is influenced by parameters α, γ, ω and the normal force P. The normal force has three critical values P_{cr2}, P_{cr4} and P_{cr6}. These sliding modes are:

Case 1. When $\beta < -\omega$, stick-slip motion occurs. In this case $\alpha < 0$, $\omega_{cr} = \omega$ and $P > P_{cr2}$, where the critical force is:

$$P_{cr2} = \frac{-2 m_s \omega_{cr} - \gamma}{\alpha}. \tag{11}$$

Limit case 2. When $\beta = -\omega$ we also have stick-slip motion. In this limit case, similar to case 1, $\alpha < 0$, $\omega_{cr} = \omega$. The only difference is that $P = P_{cr2}$.

Case 3. In this case, when $-\omega < \beta < 0$, we have driven oscillations mixed with stick-slip motion. $\alpha < 0$ and $P_{cr2} > P > P_{cr4}$, where:

$$P_{cr4} = \frac{-\gamma}{\alpha}. \tag{12}$$

Limit case 4. Here, $\beta = 0$, motion appears in the form of harmonic oscillations. In this limit case, we can have $P = P_{cr4}$ or P can have any value, if $\alpha = 0$ and $\gamma = 0$.

Case 5. For this sliding mode, when $0 < \beta < \omega$, we have harmonic damped oscillations. Here $\alpha > 0$ and $P_{cr4} < P < P_{cr6}$, where:

$$P_{cr6} = \frac{2 m_s \omega_{cr} - \gamma}{\alpha}. \tag{13}$$

Limit case 6. In this last limit case, when $\beta = \omega$, the stick-slip phenomenon disappears and we will have smooth sliding. For this case $\alpha > 0$ and $P = P_{cr6}$.

Case 7. In this final sliding mode, when $\beta > \omega$, we will also have smooth sliding. For this case $\alpha > 0$ and $P = P_{cr6}$.

We can calculate the displacement with equation (14) and the speed with equation (15):

$$x = x_0 + e^{-\beta t}\left[(x_p - x_0)\cos(\Omega t) + \frac{v_0}{\Omega}\sin(\Omega t) + \frac{\beta}{\Omega}(x_p - x_0)\sin(\Omega t)\right], \tag{14}$$

$$v = e^{-\beta t}\left[v_0\cos(\Omega t) + \frac{v_0\beta}{\Omega}\sin(\Omega t) - \left(1 + \frac{\beta^2}{\Omega^2}\right)\Omega(x_p - x_0)\sin(\Omega t)\right]. \tag{15}$$

The acceleration is:

$$a = \frac{dv}{dt}. \tag{16}$$

The kinetic friction coefficient will be:

$$\mu_k = \mu_0 + \alpha v_0 - \alpha v. \tag{17}$$

The time at which the displacement has the maximum value, hence a null speed, will be:

$$t_{max} = \frac{1}{\Omega}\operatorname{atan}\left[\frac{1}{\frac{\beta}{\Omega} + \frac{\Omega}{v_0}\left(1 + \frac{\beta^2}{\Omega^2}\right)(x_p - x_0)}\right] \tag{18}$$

The abscissa x_p corresponds to the end of the stick period and the beginning of the slip period:

$$x_p = \frac{P}{k\mu_s}. \tag{19}$$

If the static coefficient of friction μ_s is a function of idle time (slip), then x_p will be determined as a function of t.

Thus, the oscillation amplitude A_0 will be:

$$A_0 = x_{max} - x_0. \tag{20}$$

At the end of the slip period t_p the stick period begins. Corresponding to this time, the speed becomes equal with the driving speed of the fixed sample v_0:

$$v_{rel} = v - v_0. \tag{21}$$

With the testing apparatus from The Department of Machine Elements and Tribology, University "Politehnica" Bucharest, used for the current experiments, depending on the evolution of the friction coefficient with speed, we can obtain each of the seven sliding modes that are influenced by the effective damping coefficient β by modifying the normal force P between the two contact surfaces of the friction couple materials. Hereinafter, by solving the equations the theoretical model with the help of the Mathcad software, we can graphically see the evolution of the displacement for all of the seven sliding modes, for different input values.

The mass of the analyzed fixed sample is m_s = 0.643 kg, the spring constant of the measuring system is $k = 14.77 \cdot 10^3$ N/m and the critical oscillation frequency is ω_{cr} = 151.56 Hz.

For case 1, where the stick-slip phenomenon occurs, we considered the friction to be dry ($\gamma = 0$) and from preliminary experimental trials we accepted α = -0.5. For this case the normal force must be higher than the critical value $P > P_{cr2}$, where P_{cr2} = 389.812 N. In Fig. 3 we can observe the evolution of the displacement for P = 390 N and P = 420 N.

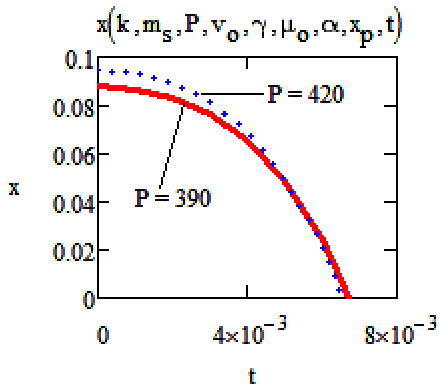

Figure 3. Evolution of displacement for case 1

In Figure 4 we can observe the evolution of the displacement for the limit case 2 when the normal force is $P = P_{cr2}$ = 389.812 N. For this limit case, where we also have stick-slip phenomenon, as in case 1 we considered the friction to be dry (γ = 0) and from preliminary experimental trials we accepted α = -0.5.

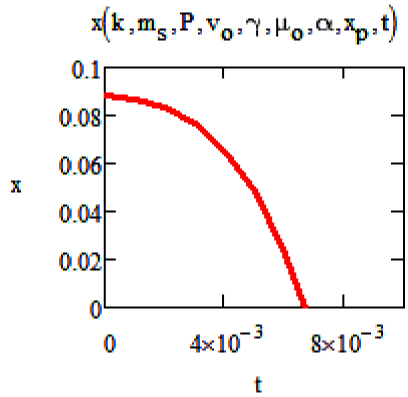

Figure 4. Evolution of displacement for limit case 2

The evolution of the displacement for case 3, where we have driven oscillations mixed with stick-slip motion, can be observed in Figure 5. For this case we also considered the friction to be dry (γ = 0) and we accepted the same α = -0.5. Here, the normal force must be between the two critical values $P_{cr2} > P > P_{cr4}$, which are P_{cr2} = 389.812 N and P_{cr4} = 0 N.

For limit case 4, where we have harmonic oscillations, we considered the friction to be dry (γ = 0) and accepted α = 0. For this case the normal force can have any value, so we compared the evolution of the displacement for P = 5 N and P = 10 N (Fig. 6).

Case 5, where we have harmonic oscillations, has the condition that the normal force must be between the two critical values $P_{cr4} < P < P_{cr6}$. For dry friction and α = 0.5, the critical values of the normal force are P_{cr4} = 0 N and P_{cr6} = 389.812 N. In Figure 7 we observe the evolution of the displacement for P = 5 N and P = 10 N.

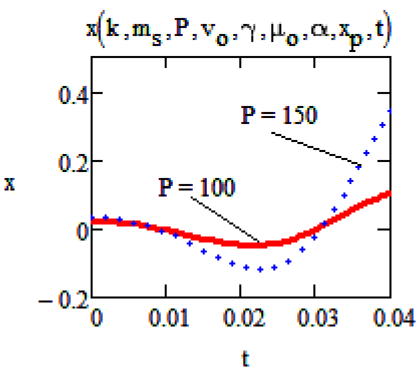

Figure 5. Evolution of displacement for case 3

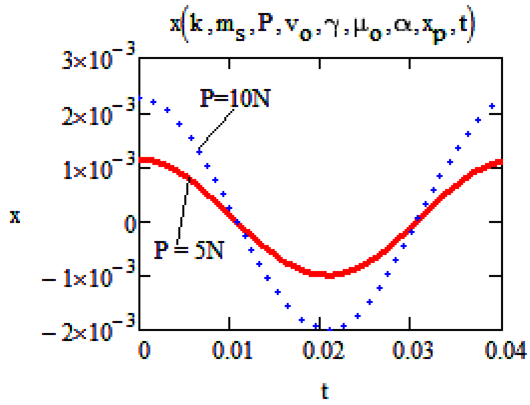

Figure 6. Evolution of displacement for limit case 4

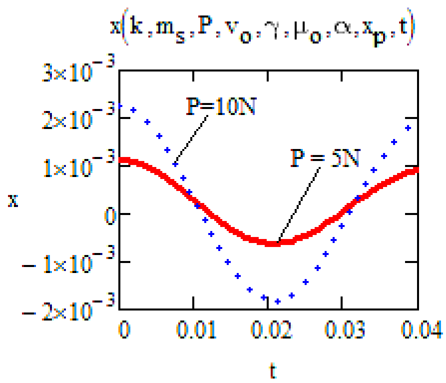

Figure 7. Evolution of displacement for case 5

In limit case 6 the stick-slip phenomenon disappears and we will have smooth sliding. For this case we considered the same dry friction (γ = 0) and α = 0.5. The normal load must have the critical value $P = P_{cr6}$. For this limit case, when P = 389.812 N, the evolution of the displacement can be observed in Figure 8.

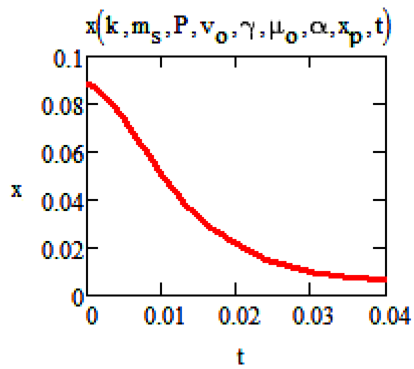

Figure 8. Evolution of displacement for limit case 6

Case 7, where the normal force must be higher than the critical value $P > P_{cr6}$, has a motion described by smooth sliding (no stick-slip). For the same conditions as the previous case (γ = 0 and α = 0.5), we can observe the evolution of the displacement for normal forces higher than the critical value P = 390 N and P = 550 N (Fig. 9).

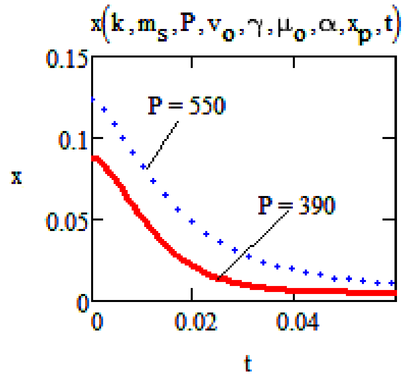

Figure 9. Evolution of displacement for case 7

4. CONCLUSIONS

The most widely accepted cause for the stick-slip phenomenon is that the values of the static friction coefficient (μ_s) exceed the ones of the kinetic friction coefficient (μ_k). It is assumed that the kinetic friction is linearly dependant on speed and the static friction is exponentially dependant on stick time.

Based on the values of the effective damping coefficient β, which can be positive, negative or zero, there can be seven different sliding modes. In these cases sliding is influenced by parameters α, γ, ω and the normal force *P*.

The theoretical model developed can help study the behaviour of the brake friction couple materials at low speeds, when stick-slip occurs.

Through the theoretical model we were able to highlight the critical force and implicitly the critical contact pressure at which the stick-slip phenomenon occurs at the specific contact of the friction materials for the automotive disc brake system.

The equations developed here can be used for the study of the experimental results obtained with the testing apparatus from our department specially designed for the study of friction and wear at low and very low sliding speeds. Future experimental work must be done to validate the theoretical model elaborated in this article.

ACKNOWLEDGEMENT

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