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THE EFFECT OF STATIC FRICTION ABOUT LOADING OF COULOMBIAN DAMPERS

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Abstract: *The limitation of the operation of conventional dry friction dampers (Coulombian dampers) is dependent on the static and kinetic friction coefficients. The paper aims to determine the values of two dampers solutions, one with a ring fixed to the outer tube and another with a ring fixed on the rod. A theoretical analysis of the rings deformation was performed for two constructive solutions. At the assembly, the ring is considered as a thick tube and in operation as an annular plate subject to tangential forces (friction forces) on the inner contour. For the theoretical model the values of the friction coefficients used were determined experimentally.*

Keywords: *Coulombian damper, friction coefficients, tangential forces, static friction.*

1. INTRODUCTION

For vibration damping at different installations are used and dry friction dampers (Coulombian damper) tube-type ring-rod [1-4]. There are constructive solutions with axial fixing of the tube through elastic ring shape tube (tube with the lobes).

The ring in the free state is a prism with a trapezoidal section with dimensions h, s_1, $L_{12} = \pi d_{1e}$, $L_{13} = \pi d_{1i}$. After mounting in the tube lobes, the ring deforms so penetrate the lobes and it generates the contact pressure on the rod. Movement or trend in the movement of the rod towards the tube, there is friction between the ring and rod. This friction dampens the vibrations on direction rod.

This paper aims to analyze the geometric conditions of ring elements (h, s_1, d_{1e}, d_{1i}) of the tube (lobes geometry Δ, λ, d_{20}), of the rod (d_{3e}, d_{3i}) and material properties, so that after mounting the ring fill wholly or partially the lobes and the global deformation response regime to be elastic.

In this sense it determines the pressure of contact between ring and tube and between the ring and rod. Effect of contact pressure on the state of deformations and stresses is analyzed on the basis of the theory of elasticity.

The contact pressure is determined by the geometry of the ring, of the tube and the elasticity characteristics of the material. The relative movement between rod and ring only appears when the force of the depreciation is greater than the force of static friction.

This force defines load-bearing capacity of the shock absorber Coulombian.

2. ANALYTICAL MODEL FOR THE CALCULATION OF CRITICAL PRESSURE

It is considered that a recess of the tube is shaped like a torus with axial section cosinusoidal (Fig. 1) .

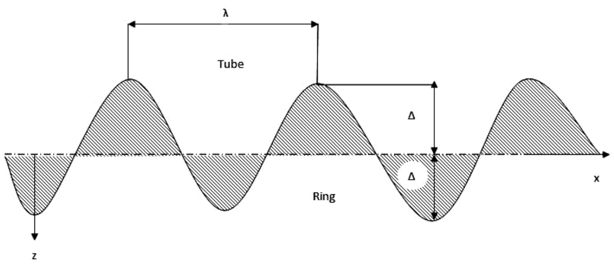

Figure 1. The tube

$$z = \Delta \cdot \cos\frac{2\pi x}{\lambda} = \Delta \cdot \cos(\alpha x) \quad (1)$$

with $\alpha = \frac{2\pi}{\lambda}$.

Unfold the ring and it is considered a contact elastic smooth plain surfaces with a profiled rigid surface (inner surface carried out of the tube). In this hypothesis it is accepted the stress function Airy on the form [6,7]:

$$\Phi_{(x,y)} = \left(\frac{p_s}{\alpha^2}\right) \cdot (1+\alpha z) e^{-\alpha z} \cdot \cos(\alpha x). \quad (2)$$

where p_s is the amplitude of the pressure of the contact and will be determined the terms and conditions outline.

In this case, the tensions from a point on the ring are:

$$\sigma_x = \frac{\partial^2 \Phi}{\partial z^2} = -p_s (1-\alpha z) e^{-\alpha z} \cdot \cos(\alpha x) \quad (3a)$$

$$\sigma_z = \frac{\partial^2 \Phi}{\partial x^2} = -p_s (1+\alpha z) e^{-\alpha z} \cdot \cos(\alpha x) \quad (3b)$$

$$\tau_{xz} = -\frac{\partial^2 \Phi}{\partial x \partial z} = p^* \alpha z e^{-\alpha z} \cdot \sin(\alpha x) \quad (3c)$$

$$\sigma_y = \vartheta (\sigma_x + \sigma_z). \quad (3d)$$

On the basis of Hooke's law it is determined the relative deformations ε_x, ε_y, ε_z, γ_{xz} and then the displacements u_x, u_y and u_z:

$$\varepsilon_x = \frac{1+\vartheta}{E}\left[(1-\vartheta)\sigma_x - \vartheta\sigma_z\right]. \quad (4a)$$

$$\varepsilon_z = \frac{1+\vartheta}{E}\left[(1-\vartheta)\sigma_z - \vartheta\sigma_x\right]. \quad (4b)$$

$$\gamma_{xz} = \frac{2(1+\vartheta)}{E}\tau_{xz}. \quad (4c)$$

$$\varepsilon_y = 0 \quad . \quad (4d)$$

$$u_x = \int \varepsilon_x dx = -p_s \frac{1+\vartheta}{E} \cdot \frac{1}{\alpha} \sin(\alpha x) e^{-\alpha z} (1 - 2\vartheta - \alpha z) + c \quad (5a)$$

$$u_z = \int \varepsilon_z dz \quad (5b)$$

$$\int \varepsilon_z dz = p_s \frac{1+\vartheta}{E} \cdot \cos(\alpha x) e^{-\alpha z} \left(\frac{2(1-\vartheta)}{\alpha} + z\right) + c \quad (5c)$$

where *c* is a constant of integration.

Contact pressure amplitude p_s and the constant c are determined in terms of the outline conditions on the deformed geometry of the ring in the lobes (the state plane of deformations) $u_z = \Delta$ and $u_x = \lambda/4$ for x = 0 and z = 0.

From (5a), (5b) and (5c) in these circumstances the limit conditions (outline conditions), result:

$$p_s = \left(\frac{\Delta}{\lambda} - \frac{1}{4}\right)\frac{\pi E}{1-\vartheta^2} \quad (6a)$$

$$c = \frac{\lambda}{4} \quad (6b)$$

Substituting these constants of integration (p_s and *c*) in (5a, b and c) it will determine the displacements at any point of the damper ring.

For contact points between the ring and tube (z = 0), result:

$$\overline{u_x} = u_x|_{z=0} = \frac{\lambda}{4}\left[1 - \frac{2(1-2\vartheta)}{1-\vartheta}\left(\frac{\Delta}{\lambda} - \frac{1}{4}\right)\sin\frac{2\pi x}{\lambda}\right] \quad (7a)$$

$$\overline{u_z} = u_z|_{z=0} = \frac{\lambda}{4}\left[1 + \left(4\frac{\Delta}{\lambda} - 1\right)\cos\alpha x\right] \quad (7b)$$

From (7) it is observed that the outline conditions : $\overline{u_x}|_{x=0} = \frac{\lambda}{4}$ and $\overline{u_z}|_{x=0} = \Delta$.

The condition concerning the filling of cosinusoidal lobes of the rigid tube, characterized by amplitude Δ and the wavelength λ (the distance between two neighbouring alveoli) is given by the pressure p^* (6a). This pressure is defined as the critical pressure.

Thus, between Δ and λ there must be the inequality:

$$\Delta \leq \frac{\lambda}{4}. \quad (8)$$

Critical pressure necessary to fill the rigid lobes of the damper Coulombian rigid tube shall be determined by the relationship (6a) , while abiding by the restriction $\frac{\Delta}{\lambda} \leq \frac{1}{4}$.

For any contact pressure lower than critical, elastic ring doesn't fills completely the lobes tube.

3. THE STRESS STATE OF THE ELASTIC RING

For geometrical optimization of Coulombian damper with rubber rings we should make an analysis of the state of tension in the ring and accepts a criterion for deteriorate of the material of the ring.

To make the following dimensional:

$$p_{as} = p_s / E; \ \sigma_{ax} = \sigma_x / E;$$
$$\sigma_{az} = \sigma_z / E; \ \sigma_{ay} = \sigma_y / E;$$
$$\tau_{axz} = \tau_{xz} / E; \ \Delta_a = \Delta / h;$$
$$\lambda_a = \lambda / h; \ h_a = 2h / d_{20};$$
$$z_a = z / \Delta; \ x_a = x / \lambda.$$

Thus obtained:

$$p_{as} = \frac{\pi}{1-\nu^2}\left(\frac{\Delta_a}{\lambda_a} - \frac{1}{4}\right) \quad (9)$$

$$\sigma_{ax} = -p_{as}\left(1 - \frac{2\pi\Delta_a}{\lambda_a} z_a\right) e^{-\frac{2\pi\Delta_a}{\lambda_a} z_a} \cdot \cos(2\pi x_a) \quad (10a)$$

$$\sigma_{az} = -p_{as}\left(1 + \frac{2\pi\Delta_a}{\lambda_a} z_a\right) e^{-\frac{2\pi\Delta_a}{\lambda_a} z_a} \cdot \cos(2\pi x_a) \quad (10b)$$

$$\sigma_{ay} = \nu(\sigma_{ax} + \sigma_{az}); \quad (10c)$$

$$\tau_{axz} = p_{a1} \frac{2\pi\Delta_a}{\lambda_a} z_a e^{-\frac{2\pi\Delta_a}{\lambda_a} z_a} \sin(2\pi x_a) \quad (10d)$$

In Figures 2 – 6 there are presented adimensional pressure critical dependencies and tensions of a damper ring Coulombian.

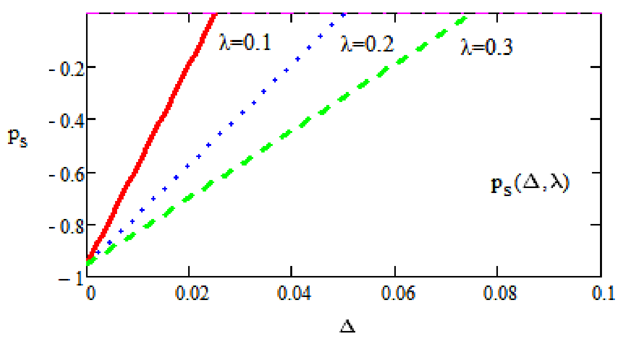

Figure 2. Pressure dependencies

On the basis of main strains:

$$\varepsilon_1 = 0.5(\varepsilon_x + \varepsilon_z) + 0.5\sqrt{(\varepsilon_x - \varepsilon_z)^2 + \gamma_{xz}^2} \quad (11a)$$

$$\varepsilon_2 = 0.5(\varepsilon_x + \varepsilon_z) - 0.5\sqrt{(\varepsilon_x - \varepsilon_z)^2 + \gamma_{xz}^2} \quad (11b)$$

determinate the main tensions adimensional:

$$\sigma_{a1} = \frac{\varepsilon_1 - \nu(\varepsilon_1 - \varepsilon_2)}{(1+\nu)(1-2\nu)}, \quad (12a)$$

$$\sigma_{a2} = \frac{\varepsilon_2 - \nu(\varepsilon_2 - \varepsilon_1)}{(1+\nu)(1-2\nu)}, \quad (12b)$$

$$\sigma_{a3} = \frac{\nu(\varepsilon_1 + \varepsilon_2)}{(1+\nu)(1-2\nu)}, \quad (12c)$$

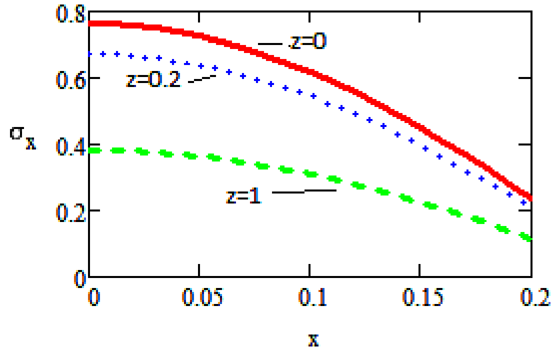

Figure 3. Tension from the damper

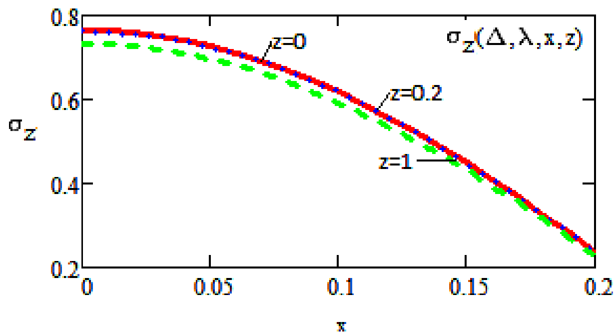

Figure 4. Tension from the damper, σ_x

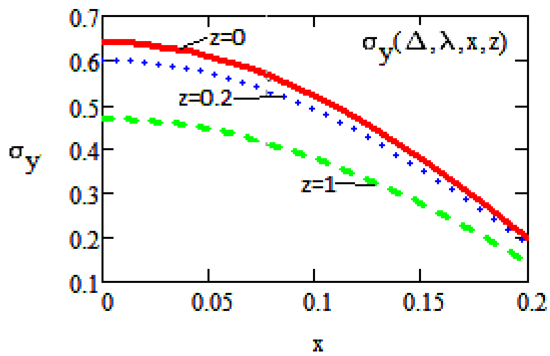

Figure 5. Tension from the damper, σ_y

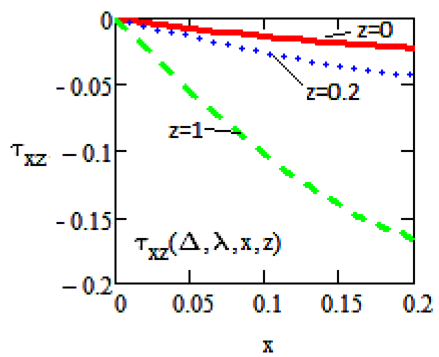

Figure 6. Tension from the damper, σ_z.

With the main tension can be determined the tension equivalent [8].

According to the theories of the $\underline{II}$-nd (the maximum specific deformation) , the $\underline{III}$-rd (tangential maximum tension), the $\overline{V}$ -th (specific potential energy variation of shape), dimensionless equivalent tension is presented in Figure 7 and 8 as a function of the position of the point from the damper's elastic ring.

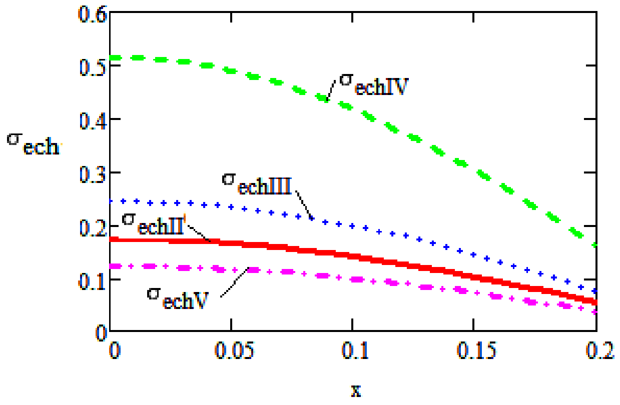

Figure 7. Equivalent tension

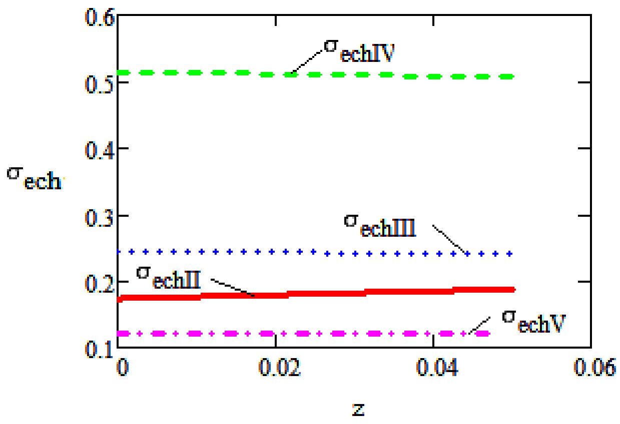

Figure 8. Equivalent tension

From the analysis of graphs and analytical study of the main tensions, it follows that the tension equivalent is maximum (z = 0; x = 0; x = λ; x = 2λ; ...; x = mλ) where m is an integer. For the assessment of equivalent maximum tension is applied to the theory of $\overline{IV}$ -th of resistance (specific energy of deformation).

$$\sigma_{a\,echIV} = \sqrt{\sigma_{a1}^2 + \sigma_{a2}^2 + \sigma_{a3}^2 - 2\nu(\sigma_{a1}\sigma_{a2} + \sigma_{a1}\sigma_{a3} + \sigma_{a2}\sigma_{a3})} \quad (13)$$

and

$$\sigma_{a\,ech} = \sigma_{a\,echIV}(z = 0, x = 0, \lambda, 2\lambda, \ldots, m\lambda) \quad (14)$$

In Figure 9 there is represented the variation of maximum adimensional equivalent tension function of lobes (Δ_a) and different wavelengths of the lobes (λ_a).

If the maximum equivalent tension from the damper's ring is smaller than a specific resistance of the material (σ_L), then the ring will deform elastically and the accumulation of energy by hysteresis is causing the removal from service.

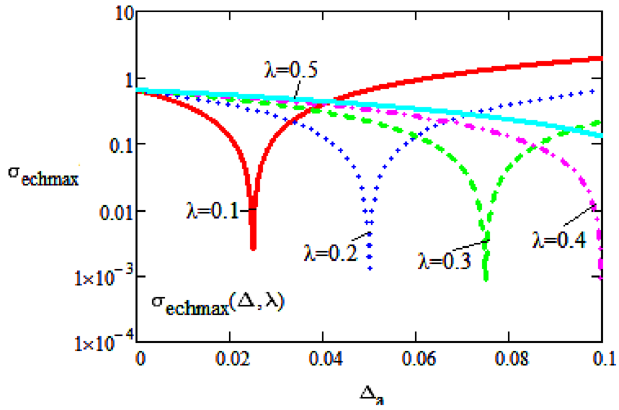

Figure 9. Variation of maximum adimensional equivalent tension

If the maximum equivalent tension is greater than the specific resistance, then the damage will be done by breaking the ring.

Taking into account that the maximum equivalent tension is adimensional through the longitudinal elasticity module (E), It means that this report represents the specific unconventional deformation (σ/E, τ_{xz}/E).

If specific resistance is reported at the longitudinal elasticity module (s), then operating under the condition of function in elastic system is expressed by specifying conventional deformation.

$$\varepsilon_{ech} = \sigma_{ech} = \frac{\sigma_{echmax}}{E} \le \varepsilon_{cr} . \quad (15)$$

where ε_{cr} is a critical relative deformation, determined by standardized tests of traction and compression.

It defines the operating parameter of the damper's ring:

$$P_{el} = \varepsilon_{ech} - \varepsilon_{cr} . \quad (16)$$

and is presented in Figure 10 .

For negative values of the parameter for the operation of the damper's ring P_{el}, the state of global deformation is elastic and for positive values, the state of deformation becomes plastic.

Depending on the values of the coefficients static friction and kinetic, between the damper's rod and the ring , based on (16) and Figure 10, to determine the maximum load transmitted from the shock absorber.

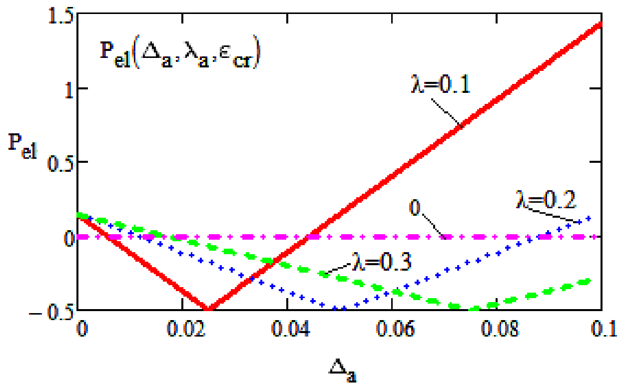

Figure 10. The operating parameter of the damper's ring

Limitation of such loads will be described in a subsequent work by determining the static and kinetic friction coefficients at different speeds.

4. CONCLUSIONS

Axial fixing of the ring in the alveoli can constitute a favourable technical solution.

The optimum geometry of the alveoli is dependent on ring's elasticity and on the diameter of the damper's rod.

Maximum equivalent tension of the ring appears on the exterior surface of the ring upon contact with the tube.

Theoretical modelling leads to equivalent maximum tension after the $\overline{IV}$ -th theory (specific energy of deformation).

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