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TRIGGER OF COUPLED SINGULARITIES IN VIBRATIONS OF THE SYSTEM WITH COULOMB'S TYPE FRICTIONS

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Abstract: *Two models of the dynamics of abstraction of the real engineering systems consisting of heavy slider slides along rotate rough curvilinear shapes guides are described. Positions of different rough surfaces on the rough curvilinear shapes guides are studied and appearance of the corresponding Amonton-Coulomb's type friction force is defined. Also, for different models of dynamics corresponding to nonlinear differential equations are presented. First group of the alternations and discontinuities of kinetic parameter properties are identified in the system with no ideal constraints caused by Amonton-Coulomb type friction. By using examples of the heavy material particle motion along rough plane or rotate rough curvilinear lines (ellipse, circle, cycloid, parabola, arbitrary curvilinear line) in vertical plane, by derived corresponding differential equations and corresponding equations of the phase trajectory series of the discontinuities of system kinetic parameter properties are identified.*

Keywords: *rotate rough curvilinear line, Amonton-Coulomb's type friction, heavy mass particle, relative pseudo-equilibrium position, trigger of coupled singularities, alternation, one side singular point, phase trajectory, homoclinic orbit.*

1. INTRODUCTION

For introduction in description of the special phenomena appearing in the oscillatory system dynamics with Amonton-Coulomb's type friction it is useful to made an analysis of the classical and much known oscillator with friction. In each classical university literature is possible to see this example of oscillator with friction (see Reference [1] by D. Rašković). This oscillator contain one heavy mass particle, mass m, coupled by a ideal elastic spring, with stiffness c, moving along horizontal rough plane with coefficient of friction μ (Fig. 1a). By analysis of kinetic parameters of this oscillator with friction, known from References [1,2], it is possible to point out new conclusion. On the basic, that a nonlinear phenomenon is observable in a set of characteristic positions in dynamics of considered oscillator with friction. A trigger [3-6] of coupled two one side singularities and one, fictive singular point between them exists in the phase portrait. Two, one side singular points S_1 and S_2 around equilibrium position S_0 corresponding to equivalent oscillator (as conservative linear system) without friction, are at distance of $s=\mu\frac{g}{\omega_0^2}$, where $\omega_0^2=\frac{c}{m}$. These three points $S_1: x=-s=-\mu\frac{g}{\omega_0^2}$, $S_0: x=0$, and $S_2: x=s=\mu\frac{g}{\omega_0^2}$ represent a trigger of coupled one side (half) singular points [1].

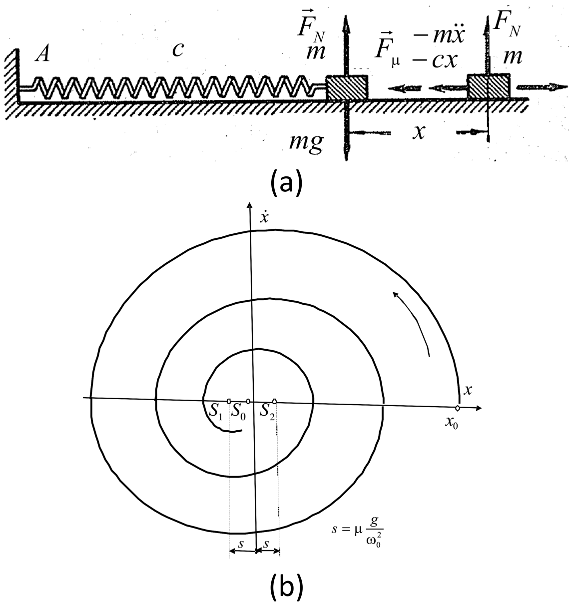


Figure 1. (a) classical oscillator with friction with plan of active and reactive forces and (b) phase trajectory portrait of dynamics oscillator with friction

Double differential equation of dynamics of classical oscillator with friction is in the form:

$$m\ddot{x} + cx = \mp \mu mg \begin{cases} \dot{x} > 0 \\ \dot{x} > 0 \end{cases}$$

or

$$\ddot{x} + \omega_0^2 x = \mp \mu g \begin{cases} \dot{x} > 0 \\ \dot{x} > 0 \end{cases} \quad (1)$$

Double equation of the phase trajectory of classical oscillator with friction is in the form:

$$\dot{x}^2 = \dot{x}_0^2 - \omega_0^2 \left[(x \pm s)^2 - s^2 \mp 2sx_0 \right] \begin{cases} \dot{x} > 0 \\ \dot{x} > 0 \end{cases} \quad (2)$$

or in the form

$$\frac{(x \pm s)^2}{\left(\frac{\dot{x}_0^2}{\omega_0^2} + s^2 \pm 2sx_0 \right)} + \frac{\dot{x}^2}{\omega_0^2 \left(\frac{\dot{x}_0^2}{\omega_0^2} + s^2 \pm 2sx_0 \right)} = 1 \begin{cases} \dot{x} > 0 \\ \dot{x} > 0 \end{cases} \quad (3)$$

From last form of the transformed double equation od the phase trajectory of dynamics of classical oscillator with friction, it is visible that branch od the phase trajectory are graph branches of ellipses with alternation in the cenre $x = \mp s, \dot{x} = 0$, and half axis equal to:

$$a = \sqrt{\frac{\dot{x}_0^2}{\omega_0^2} + s^2 \pm 2sx_0}, \quad for \begin{cases} \dot{x} > 0 \\ \dot{x} > 0 \end{cases} \quad (4)$$

$$b = \omega_0 \sqrt{\frac{\dot{x}_0^2}{\omega_0^2} + s^2 \pm 2sx_0} = \omega_0 a, \quad for \begin{cases} \dot{x} > 0 \\ \dot{x} > 0 \end{cases} \quad (5)$$

in alternations depending of direction of the mass motion, and alternation of the friction force and velocity direction.

In this system, energy dissipation appear and equal to power of friction force along heavy mass particle path in each interval of the mass particle velocity and friction force alternation of directions:

$$\frac{d\mathbf{E}_{total}}{dt} = \frac{d(\mathbf{E}_k + \mathbf{E}_p)}{dt} = \mathbf{P}_{\mathbf{F}_\mu} = \mp F_{\varpi} \dot{x} \quad \begin{cases} \dot{x} > 0 \\ \dot{x} > 0 \end{cases} \quad (6)$$

or for finite intervals between alternation of the friction force directions:

$$\mathbf{E}_{total} - \mathbf{E}_{0,total} = \mathbf{E}_k + \mathbf{E}_p - (\mathbf{E}_{0,k} + \mathbf{E}_{0,p}) = \mp F_{\varpi}(x - x_0) \begin{cases} \dot{x} > 0 \\ \dot{x} > 0 \end{cases} \quad (7)$$

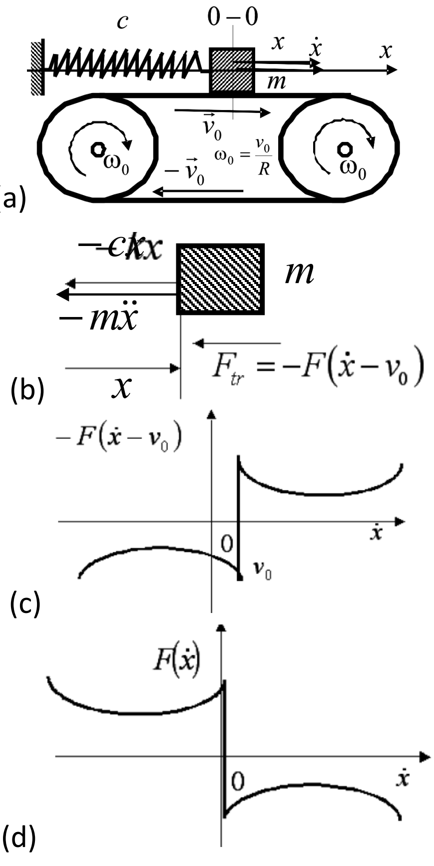


Figure 2. (a) a model of oscillator with dray friction at axially moving rough belt, (b) plan of forces in direction of mass motion, (c) and (d) graphic of friction force depending of absolute and relative velocity of the mass particle in relation of axially moving belt

If heavy mass particle moves, along moving rough plane then oscillator with friction is strong nonlinear dynamical system. Real system of like that model of oscillator with friction is presented at Figure 2 [2].

In Figure 2a, a model of oscillator with dray friction at axially moving rough beltm with velocity v_0 is presented. In Figure 2b is possible to see corresponding plan of forces in direction of mass motion including friction force depending of heavy mass particle relative velocity in relation to axially moving belt. Graphic of friction force dependence of absolute (Fig. 2d) and relative velocity (Fig. 2c) of the mass particle in relation of axially moving belt are presented.

Nonlinear dynamics of oscillator with friction on the axially moving belt, presented in Figure 2a is describing by non-linear Van der Pol differential equation, with large parameter ε:

$$\frac{d^2x}{dt^2}+\varepsilon\left[\frac{dx}{dt}-\frac{1}{3}\left(\frac{dx}{dt}\right)^3\right]+x=0 \qquad (7)$$

or in general form:

$$\frac{d^2x}{dt^2}+\varepsilon f\left(\frac{dx}{dt}\right)+x=0 \qquad (8)$$

where friction force, depending of absolute velocity in nonlinear relation, is in the form:

$$-F(\dot{x})=\varepsilon\left[\frac{dx}{dt}-\frac{1}{3}\left(\frac{dx}{dt}\right)^3\right] \qquad (9)$$

or in general non-linear function form:

$$-F(\dot{x})=\varepsilon f\left[\frac{dx}{dt}\right] \qquad (10)$$

2. A REAL CONSTRUCTION SYSTEM DYNAMICS WITH FRICTION

Let's consider dynamics of a model abstraction of real construction system dynamics with friction between a slider, mass m, coupled by a ideal elastic spring with stiffness c, for axis of rotation, slides along horizontal rough lath rotate around vertical axis, with velocity $\vec{\Omega}$. We take into account two case of lath cross section: 1* rectangular cross section of rough rotate lath with all four rough planes, two parallel with coefficient of friction μ_1 and two other with coefficient of friction μ_2; 2* cross section of rough rotate lath is circle and with cylindrical rough surface with coefficient of friction μ.

For first case we can consider a slider slides along rough lath with rectangular cross section (Fig. 3). Between surfaces of slider and rough lath, during the relative motion, appear two components of Amonton-Coulomb's type friction forces. These components of the friction force depends of the intensity of the corresponding force of pressers between these contact surfaces in relative motions and depend of directions of this relative velocity between them, which alternates, depending of direction of motion and direction of relative velocity.

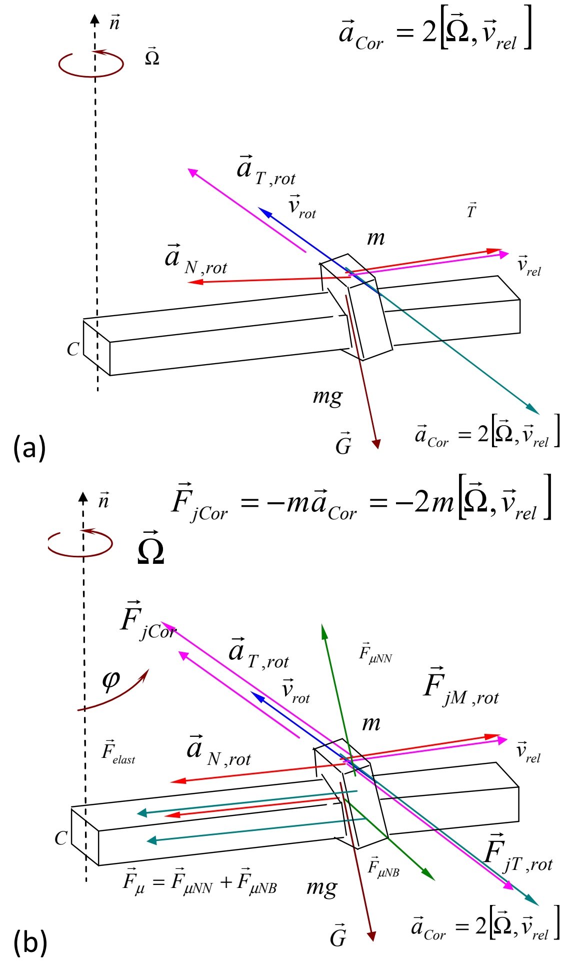


Figure 3. A slider slides along rough lath with rectangular cross section: (a) component accelerations of the slider and (b) component forces of inertia of moving slider

For obtaining interactions between rough surfaces of slider and lath, it is necessary to compose governing equation of dynamic equilibrium of the system. In Figure 3a component accelerations of the slider are presented including a component of Coriolis acceleration. In Figure 3b component forces of inertia of slider are presented.

System of differential equations of dynamic equilibrium of slider and of model is in the following form:

$$\left[\mathbf{J}+(r_0+s)^2 m\right]\ddot{\varphi} \approx \approx \begin{cases} 0 \; for\ free\quad rotation \\ \mathbf{M}\; for\ free\ or\ programed\ rotation \end{cases} \quad (11)$$

$$(r_0+s)^2 m\ddot{\varphi} + 2m(r_0+s)\dot{\varphi}\dot{s} + F_{NB} = 0 \quad (12)$$

$$m\ddot{s} - (r_0+s)m\dot{\varphi}^2 + 2m(r_0+s)\dot{\varphi}\dot{s} + cs \pm \pm\mu_1 mg \pm \mu_2 F_{NB} = 0,\ for \begin{cases}\dot{s}>0\\ \dot{s}<0\end{cases} \quad (13)$$

where we denote with φ angle of lath rotation, s coordinate of relative motion slider along lath measured from equilibrium position at distance r_0 in relation of rotation axis, taking into account that angle φ and s are generalized coordinates in one case, and $\mathbf{J}$ is axial mass inertia moment of lath for axis of lath rotation. In other case that s s generalized coordinate and φ is known rheonomic coordinate [7-9].

For general case of the considered model, ordinary differential equation of the slider relative motion along rough lath, in the approximate form, is:

$$m\ddot{s}^2 + 2\dot{s}m(r_0+s)(1\pm\mu_2)\left\langle \dot{\varphi}_0 + \frac{1}{\left[\mathbf{J}+(r_0+s)^2 m\right]}\int_0^t \mathbf{M}(t)dt \right\rangle + cs \approx \mp\mu_1 mg + \mu_2\left\{\frac{(r_0+s)^2 m}{\left[\mathbf{J}+(r_0+s)^2 m\right]}\mathbf{M}(t)\right\} - (r_0+s)m\left\langle \dot{\varphi}_0 + \frac{1}{\left[\mathbf{J}+(r_0+s)^2 m\right]}\int_0^t \mathbf{M}(t)dt \right\rangle^2, for \begin{cases}\dot{s}>0\\ \dot{s}<0\end{cases} \quad (14)$$

and binormal force of pressure between slider and lath, in approximate form, is:

$$F_{NB} \approx \frac{(r_0+s)^2 m}{\left[\mathbf{J}+(r_0+s)^2 m\right]}\mathbf{M}(t) + 2m(r_0+s)\dot{s}\left\langle \dot{\varphi}_0 + \frac{1}{\left[\mathbf{J}+(r_0+s)^2 m\right]}\int_0^t \mathbf{M}(t)dt \right\rangle \quad (15)$$

and lath angular velocity , in approximate form, is in the form:

$$\dot{\varphi}(t) \approx \dot{\varphi}_0 + \frac{1}{\left[\mathbf{J}+(r_0+s)^2 m\right]}\int_0^t \mathbf{M}(t)dt \quad (16)$$

For special case that lath rotate with constant angular velocity:

$$m\ddot{s}^2 + 2(1\pm\mu_2)\dot{s}m\Omega_0(r_0+s) + cs = = \mp\mu_1 mg - (r_0+s)m\Omega_0^2,\quad for\begin{cases}\dot{s}>0\\ \dot{s}<0\end{cases} \quad (17)$$

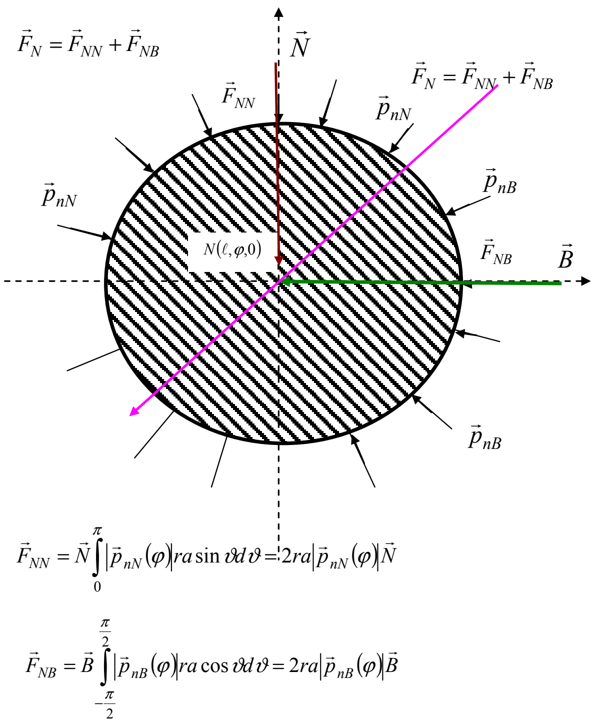


Figure 4. Plan of interaction pressures between slider and lath with cross section in the form circle

In Figure 4 plan of the interaction pressures between cylindrical contact rough surfaces of slider and lath with cross section in the form circle are presented. It is possible to see two orthogonal resultant forces of distributed interaction pressures distributed along half cylindrical contact rough surfaces, one on the horizontal base, and second on the vertical base section along lath axis [10].

Let consider a construction of the rough lath with circular cross section, presented in Figure 4, with corresponding plan of the distribution of the pressure along cylindrical surface in results of active and reactive forces between them. For this construction, we propose that relation between normal and binormal constraint reactions and corresponding pressures to the rough surfaces of the two part of half cylindrical boundary contact surface are:

$$\vec{F}_{NN} = \vec{N}\int_0^{\pi}\left|\vec{p}_{nN}(\varphi)\right| ra \sin\vartheta d\vartheta = 2ra\left|\vec{p}_{nN}(\varphi)\right|\vec{N} \quad (18)$$

$$\vec{F}_{NB} = \vec{B}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left|\vec{p}_{nB}(\varphi)\right| ra \cos\vartheta d\vartheta = 2ra\left|\vec{p}_{nB}(\varphi)\right|\vec{B} \quad (19)$$

Normal pressures distributed along contour cylindrical surface of the "rough cylindrical lath (rod)" in the form of the circular cross section with radius r , and when contact length between mass particle (slider) and half part of rough lath cylindrical surface in real construction are a :

a* in result of normal forces to the lath (axial-parallel to axis of rotation) direction, distributed pressure is :

$$\vec{p}_{nN}(\varphi) = \frac{1}{2ra}\left|\vec{F}_{NN}(\varphi)\right|\vec{n}\ ,\ \vartheta \in (0,\pi) \quad (20)$$

b* in result of the forces in binormal (circylar) direction, distributed pressure is:

$$\vec{p}_{nB}(\varphi) = \frac{1}{2ra}\left|\vec{F}_{NB}(\varphi)\right|\vec{n}\ ,\ \vartheta \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right) \quad (21)$$

Coulomb's type friction force consists from the following components:

a* in result of normal forces to the lat (axial-parallel to axis of rotation) direction resultant of friction forces is :

$$F_{nN\mu} = -\mu\int_0^{\pi}\left|\vec{p}_{nN}(\varphi)\right| ard\vartheta sign\vec{v}_{rel} = -\mu\int_0^{\pi}\frac{1}{2ra}\left|\vec{F}_{NN}(\varphi)\right| ard\vartheta sign\vec{v}_{rel}$$

$$F_{nN\mu} = -\mu\left|\vec{F}_{NN}(\varphi)\right|\frac{\pi}{2} sign\vec{v}_{rel}$$

$$\vartheta \in (0,\pi) \quad (22)$$

b* in result of the forces in binormal (circylar) direction, resultant of friction forces is :

$$F_{nB\mu} = -\mu\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left|\vec{p}_{nB}(\varphi)\right| ard\vartheta sign\vec{v}_{rel} = -\mu\int_0^{\pi}\frac{1}{2ra}\left|\vec{F}_{NB}(\varphi)\right| ard\vartheta sign\vec{v}_{rel}\ ,$$

$$F_{nB\mu} = -\mu\left|\vec{F}_{NB}(\varphi)\right|\frac{\pi}{2} sign\vec{v}_{rel}$$

$$\vartheta \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right) \quad (23)$$

Resultant of the both components of Amonton-Coulomb's type friction force is expressed by following expression:

$$F_{\mu} = F_{nN\mu} + F_{nB\mu} = -\mu\left|\vec{F}_{NN}(\varphi)\right|\frac{\pi}{2} sign\vec{v}_{rel}$$

$$-\mu\left|\vec{F}_{NB}(\varphi)\right|\frac{\pi}{2} sign\vec{v}_{rel} \quad (24)$$

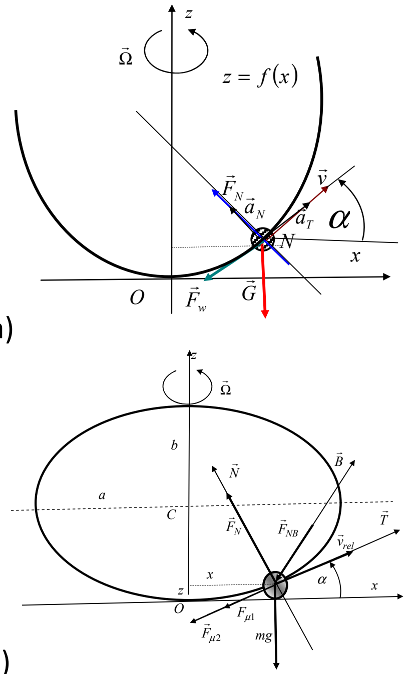
$$F_{\mu} = -\mu\left(\left|\vec{F}_{NN}(\varphi)\right| + \left|\vec{F}_{NB}(\varphi)\right|\right)\frac{\pi}{2} sign\vec{v}_{rel}$$

In comparison of resultant of Amonton-Coulomb's type friction force for two types of contact rough surfaces between heavy mass particle (slider) and rough line (lath) it is possible to made that lager friction force appear when contact surface is cylindrical them in two orthogonal contact planes, or for the case when cross section of the lath is circle them rectangular. It is visible from expression (24).

3. FREE VIBRATIONS OF THE HEAVY MASS PARTICLE ALONG ROTATE ROUGH CURVILINEAR LINE WITH COULOMB FRICTION

Let's consider free vibrations of the heavy mass particle (slider) along rotate rough curvilinear line with Amonton-Coulomb's type friction. For the case that curvilinear line is in the vertical rotate plane Oxz around vertical Oz axis, we can take that equation of the curve-linear line is: $z = f(x)$, or $f_1(x,z) = z - f(x) = 0$ and with the following properties $f(-x) = f(x)$ and that coordinate pole is in the zero point $f(0) = 0$ in which line have minimum (Fig. 5a). Also we take that curvilinear line rotate around vertical Oz axis with constant angular velocity $\vec{\Omega} = \Omega\vec{k}$.

Heavy mass particle (slider), mass m , moving along rough curvilinear line with Amonton-Coulomb's type sliding friction coefficient μ , is loaded by proper weight mg , as a active conservative force and by four no ideal constraint reactions, one F_N - normal ideal constrain reaction, second F_{BN} in binormal direction and two additional, $F_{\mu 1}$ first tangential component of the no ideal constraint reaction induced by friction and proportional to the normal component reaction F_N , $F_{\mu 1} = -\mu F_N \, sign \, \vec{v}_{rel}$, and $F_{\mu 2}$ second tangential component of the no ideal constraint reaction induced by friction caused by pressures in the binormal direction and proportional to the binormal component of the inertia force F_{BN} , $F_{\mu 2} = -\mu F_{BN} \, sign \, \vec{v}_{rel}$, caused by curvelinear line rotation around vertical Oz axis with constant angular velocity $\vec{\Omega} = \Omega \vec{k}$,



(a)

(b)

Figure 5. Heavy material particle (slider) motion: (a) along rough curvelinear line with Coulomb friction and (b) along rough ellipse line with Coulomb friction

Force of the inertia of mass particle relative motion along the curvilinear line which rotate around vertical Oz axis with constant angular velocity $\vec{\Omega} = \Omega \vec{k}$, have two components. One component is force of the inertia of the circle rotation around vertical axis in the form $\vec{F}_{jp} = m\Omega^2 x \vec{u}$, and second is Coriolis inertia force of the system and we can write: $\vec{F}_{NB} = -\vec{F}_{jC} = 2m\left[\vec{\Omega}, \vec{v}_{rel}\right] = 2m\Omega v_{rel} \cos \alpha \vec{B}$. Corresponding force of Coulomb's type friction is ubthe form:

$$\vec{F}_{\mu 2} = -\mu\left|\vec{F}_{NB}\right| \frac{\vec{v}_{rel}}{\left|\vec{v}_{rel}\right|} = -2\mu m\Omega v_{rel} \cos \alpha \frac{\vec{v}_{rel}}{\left|\vec{v}_{rel}\right|} = -2\mu m\Omega \vec{v}_{rel} \cos \alpha \quad (25)$$

Force of mass particle inertia have two components of relative inertia force and three additional components caused by curvelinear rotation around vertical axis by constant velocity, one in tangential and second in normal directions, and third in binormal direction, then we can write:

$$\vec{I}_F = -m\vec{a}_{Nrel} - m\vec{a}_{Trel} - m\vec{a}_{NPl} - m\vec{a}_{TPl} - m\vec{a}_C = -m\dot{v}_{rel}\vec{T} - m\frac{v_{rel}^2}{R_k}\vec{N} - m\Omega^2 x \vec{u} - 2m\Omega v_{rel} \cos \alpha \vec{B}$$

$$\vec{I}_F = -m\left(\dot{v}_{rel} + \Omega^2 x \cos \alpha\right)\vec{T} - m\left(\frac{v_{rel}^2}{R_k} - \Omega^2 x \sin \alpha\right)\vec{N} - 2m\Omega v_{rel} \cos \alpha \vec{B} \quad (26)$$

where R_k radius of the relative path line curvature at the point of the mass particle terminate position. $\vec{T}$, $\vec{N}$ and $\vec{B}$ are unit vectors of the tangent, normal and binormal direction to the curvelinear liner, or to the relative path line at terminate position of the mass particle during their relative motion along rough curvilinear line (for detail see Refs. [10,16]).

Final form of ordinary differential double non-linear equations of the heavy mass particle (slider) motion along rotate arbitrary curvilinear rough line, with constant angular velocity of rotation $\vec{\Omega}$, and defined by function $z = f(x)$, for the case yhat the coefficient of the Amonton-Coulomb's type sliding friction is μ , is in the following form:

$$\frac{d}{dt}\left(\dot{x}\sqrt{1+z'^2}\right) \pm \mu \dot{x}^2 \frac{z''}{\sqrt{1+z'^2}} + \Omega^2 \frac{x}{\sqrt{1+z'^2}}\left(1 \mp \mu z'\right) + \frac{g}{\sqrt{1+z'^2}}\left(z' \pm \mu\right) \pm 2\mu\Omega\dot{x} = 0 \quad (27)$$

For the case of heavy mass particle motion along ideal arbitrary curvelinear line constraint which rotate around vertical axis with constant angular velocity, differential equation is in the form:

$$\frac{d}{dt}\left(\dot{x}\sqrt{1+z'^2}\right) + \Omega^2 \frac{x}{\sqrt{1+z'^2}} + \frac{gz'}{\sqrt{1+z'^2}} = 0 \quad (28)$$

For the case of heavy mass particle motion along no ideal arbitrary rough curvilinear line without rotation, differential equation is in the form:

$$\frac{d}{dt}\left(\dot{x}\sqrt{1+z'^2}\right)+g\frac{z'}{\sqrt{1+z'^2}}\pm \mu\frac{1}{\sqrt{1+z'^2}}\left(\dot{x}^2 z''+g\right)=0 \qquad (29)$$

and normal, binormal and tangential components of the no ideal constraint rotate rough curvilinear line reaction are in the following forms:

$$F_N=\frac{m}{\sqrt{1+z'^2}}\left(\dot{x}^2 z''-\Omega^2 x z'+g\right) \qquad (30)$$

$$F_{NB}=2m\Omega\dot{x} \qquad (31)$$

$$F_{\mu 1}=-\mu\frac{m}{\sqrt{1+z'^2}}\left(\dot{x}^2 z''-\Omega^2 x z'+g\right) \qquad (30^*)$$

$$F_{\mu 2}=-2\mu m\Omega\dot{x} \qquad (31^*)$$

Finally, we can obtain a first integral of the governing equation (27), by introduce new variable in the following form: $u=\dot{x}^2$, when previous differential double equation (27) of the mass particle (slider) motion along rough arbitrary curvilinear line is transforming in the following form:

$$\frac{du}{dx}+2u\frac{z''(z'\pm\mu)}{(1+z'^2)}=-2\Omega^2\frac{x}{(1+z'^2)}(1\mp\mu z')-\frac{2g}{(1+z'^2)}(z'\pm\mu)\quad \begin{cases} for & v_{rel}>0\\ for & v_{rel}<0\end{cases} \qquad (32)$$

Previous differential double equation (32) of the material particle (slider) motion along rough curvilinear line according new helping coordinate u is ordinary differential equation first order with changeable coefficients and type in following form: $\frac{du}{dx}\pm P(x)u=Q(x)$, and is possible to obtain first integral, and for square od relative velocity of heavy slider motion along rough curvilinear line obtain in the following form:

$$v_{rel}^2(x)=(1+z'^2)e^{-2\int\frac{z''(z'\pm\mu)}{(1+z'^2)}dx}\left[-2\int\left[\Omega^2\frac{x}{(1+z'^2)}(1\mp\mu z')+\frac{g}{(1+z'^2)}(z'\pm\mu)\right]e^{2\int\frac{z''(z'\pm\mu)}{(1+z'^2)}dx}dx+C\right]\quad \begin{cases} for & v_{rel}>0\\ for & v_{rel}<0\end{cases} \qquad (33)$$

where C integral constant depending of initial conditions, angular coordinate and angular velocity at initial moment, or starting terminate mass particle positions for next phase trajectory branch.

Previous expression present double equation of the phase trajectory in phase plane of the slider nonlinear dynamics.

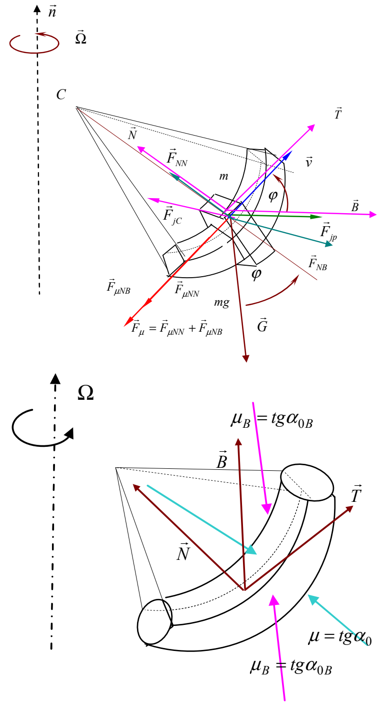


Figure 6. Two types of the construction design of curvilinear rough line with two different cross sections: (a) rectangular and (b) circle cross section

In Figure 6. two types of the construction design of curvilinear rough line with two different cross sections: (a) rectangular and (b) circle cross section are presented. Based on the analysis of the influence of lath cross sections to the resultant of the friction force from Chapter 2, using mathematical and qualitative analogies is possible to conclude that all derived ordinary double equations and double equation of phase trajectory for nonlinear motion of the slider (heavy mass particle along curvilinear line are valid for both design of curvilinear rough line with cross section rectangular and circle presented in Figure 6.

4. CONCLUSION

For the reason to compare properties of kinetic parameters and nonlinear phenomena of main considered system dynamics of a heavy slider along rough curvilinear line we use a corresponding fictive system. This fictive corresponding system is useful for comparison positions of the one side singular points and existence of trigger of couled singularities. For that reason we transform corresponding differential double equations in the forms of the system of first order differential double equations. For obtaining singularities for main system and fictive systems it is necessary to use conditions rhat right hand side all equations must be equal to zero (null). Then we obtain the following conditions [10]:

*For maun system dynamics:

$$\frac{dx}{dt} = v = 0$$
$$\frac{dv}{dt} = -\dot{x}^2 \frac{z'z''}{\left(1+z'^2\right)} \mp \mu\dot{x}^2 \frac{z''}{\left(1+z'^2\right)} - \Omega^2 \frac{x(1 \mp \mu z')}{\left(1+z'^2\right)} - \frac{g(z' \pm \mu)}{\left(1+z'^2\right)} = 0 \tag{33}$$

*For corresponding fictive systems

$$\frac{dx}{dt} = v = 0$$
$$\frac{dv}{dt} = -\Omega^2 \frac{x(1 \mp \mu z')}{\left(1+z'^2\right)} - \frac{g(z' \pm \mu)}{\left(1+z'^2\right)} = 0 \tag{34}$$

and

$$\frac{dx}{dt} = v = 0\,,\ \frac{dv}{dt} = -\frac{\left(\Omega^2 x + gz'\right)}{\left(1+z'^2\right)} = 0 \tag{35}$$

We can see that from listed systems of differential equations, of main system dynamics and first fictive system dynamics conditions for obtaining singularities are same. Depending of the curvelinear line form $z = f(x)$, we obtained two nonlinear algebra equation s in the following forms:

$$-\Omega^2 \frac{x(1 \mp \mu z')}{\left(1+z'^2\right)} - \frac{g(z' \pm \mu)}{\left(1+z'^2\right)} = 0 \text{ and } \frac{\left(\Omega^2 x + gz'\right)}{\left(1+z'^2\right)} = 0 \tag{36}$$

from which we can obtain, one or more roots.

If corresponding algebra double equation (36) have one root for $\mu = 0$ then words are about one equilibrium position with „one side left“ and „one side right“ bifurcation of the equilibrium position and one fictive trigger of coupled singularities caused by Coulomb's type friction between mass particle and rough curvelinear line.

If corresponding algebra double equation (36) have odd number of roots for $\mu = 0$ then words are about trigger of coupled singularities in a dynamics of a basic non-linear system correspond to the system with friction.

In this case corresponding algebra double equation (36) for $\mu \neq 0$ have corresponding odd number of roots for each sets of the sign $\pm$, but all these roots are slected in two subsets, first one an "one side right" singularities and other "one side left" singularities correspond to the "one side left" and "one side right" equilibrium positions. Then, each roots of the corresponding algebra double equation (36) for $\mu = 0$, have two corresponding roots obtained from corresponding algebra double equation (36) for $\mu \neq 0$ and then there are present new fictive triggers of coupled one side singularities. Then we have trigger of the coupled triggers of coupled one side left, one central and one side right singularities, which are present in the system with Amonton-Coulomb's type friction and with a corresponding nonlinear system with ideal constraints and with minimum a trigger of coupled singularities in its nonlinear dynamics.

Let to consider four examples of heavy mass particle (heavy slider) non0lunear dynamics along rough curvelinear line rotate with constant angular velocity $\vec{\Omega} = \Omega\vec{k}$, for particular choose of the rough cirvelinear line forms: *parabola $z = \frac{x^{2n}}{a^{2n-1}}$, *parabola $z = \pm\frac{x^{2n+1}}{a^{2n}}$, *circle $(z-R)^2 + x^2 = R^2$, $z = R - \sqrt{R^2 - x^2}$ and *ellipse $\left(\frac{z-R}{a}\right)^2 + \frac{x^2}{b^2} = 1$, $z = R \pm a\sqrt{1 - \frac{x^2}{b^2}}$.

a* For the parabola shaped by $z = \frac{x^{2n}}{a^{2n-1}}, n \in N$, for $\mu = 0$ and $\mu \neq 0$, from (36) there are two corresponding algebra equations, one of which for $\mu \neq 0$ is algebra double equations:

$$\Omega^2 x + g2n\frac{x^{2n-1}}{a^{2n-1}} = 0 \text{ and}$$

$$\Omega^2 x\left(1 \mp 2\mu\frac{x^{2n-1}}{a^{2n-1}}\right) + g\left(2\frac{x^{2n-1}}{a^{2n-1}} \pm \mu\right) = 0 \qquad (37)$$

From first algebra equation for $\mu = 0$ of previous system is visible that $x = 0$ is unique root correspond to the equilibrium position, and that second algebra equation for $\mu \neq 0$ have minimum two roots, or more. First approximation of the vales of two roots are $x_{1,3} \approx \pm\mu\frac{g}{\Omega^2}$ and with $x = 0$ build a trigger of coupled singularities appears as a result of bifurcation by introducing Amonton-Coulomb's type friction.

b* For the parabola shaped by $z = \pm\frac{x^{2n+1}}{a^{2n}}$, $n \in N$, for $\mu = 0$ and $\mu \neq 0$, from (36) there are two corresponding algebra equations, one of which for $\mu \neq 0$ is algebra double equations:

$$\Omega^2 x \pm (2n+1)g\frac{x^{2n}}{a^{2n}} = 0 \text{ and}$$

$$\Omega^2 x\left(1 - \mu(2n+1)\frac{x^{2n}}{a^{2n}}\right) + g\left(\pm(2n+1)\frac{x^{2n}}{a^{2n}} \pm \mu\right) = 0 \qquad (38)$$

From first algebra equation for $\mu = 0$ of previous system is visible that $x = 0$ is a root correspond to the equilibrium position, but there are also pair of the roots: $x_{1\,3} = {}^{(2n)-1}\sqrt{\pm\frac{\Omega^2 a^{2n}}{g(2n+1)}}$. In this case for $\mu = 0$ in system dynamics exists minimum a trigger of couped three singularities. Also, we can conclude that second algebra equation for $\mu \neq 0$ have minimum two roots, or more. First approximation of the minimum vales of first two roots are $x_{1,3} \approx \pm\mu\frac{g}{\Omega^2}$, which correspond to the "one side right" and "one side left" equilibrium positions and with $x = 0$ build a trigger of coupled two one side singularities appeared as a result of bifurcation by introducing Amonton-Coulomb's type friction. By qualitative analyzing of the second algebra double equation from system (38) we conclude that appear also one trigger of coupled triggers of coupled one side singularities.

c* for the circle shaped by $(z-R)^2 + x^2 = R^2$, $z = R - \sqrt{R^2 - x^2}$, for $\mu = 0$ and $\mu \neq 0$, from (36) there are two corresponding algebra equations, one of which for $\mu \neq 0$ is algebra double equations:

$$\Omega^2 x + g\frac{x}{\sqrt{R^2 - x^2}} = 0 \text{ and}$$

$$\Omega^2 x\left(1 \mp \frac{\mu x}{\sqrt{R^2 - x^2}}\right) + g\left(\frac{x}{\sqrt{R^2 - x^2}} \pm \mu\right) = 0 \qquad (39)$$

From previous first algebra equation for $\mu = 0$ of previous system (39) is visible that $x = 0$ is a root correspond to the equilibrium position, but there are also pair of the roots: $x_{1,3} = \pm\sqrt{R^2 - \frac{g^2}{\Omega^4}}$ for $\left|\frac{g}{R\Omega^2}\right| \leq 1$. In this case for $\mu = 0$ in system dynamics exists minimum a trigger of couped three singularities. Also, we can conclude that second algebra equation for $\mu \neq 0$ have minimum two roots. First approximation of the minimum vales of first two roots are $x_{1,3} \approx \pm 2\mu\frac{g}{\Omega^2}$, which correspond to the "one side right" and "one side left" equilibrium positions and with $x = 0$ build a trigger of coupled two one side singularities appeared as a result of bifurcation by introducing Amonton-Coulomb's type friction. By qualitative analyzing of the second algebra double equation from system (39) in the form:

$$x^4(1+\mu^2) - x^2\left(R^2 - (1+4\mu^2)\frac{g^2}{\Omega^4}\right) - x\left(\pm 2\mu\frac{gR^2}{\Omega^2}\right) - 4\mu^2\frac{g^2R^2}{\Omega^4} = 0$$

we conclude that appear also one trigger of coupled three triggers of coupled one side singularities.

d* for the ellipse shaped by $\left(\frac{z-R}{a}\right)^2 + \frac{x^2}{b^2} = 1$, $z = R \pm a\sqrt{1 - \frac{x^2}{b^2}}$, for $\mu = 0$ and $\mu \neq 0$, from (36) there are two corresponding algebra equations, one of which for $\mu \neq 0$ is algebra double equations (Fig. 1b):

$$\Omega^2 x \pm ag\frac{\frac{x}{b^2}}{\sqrt{1 - \frac{x^2}{b^2}}} = 0 \text{ and}$$

$$\Omega^2 x\left(1-\mu a\frac{\frac{x}{b^2}}{\sqrt{1-\frac{x^2}{b^2}}}\right)+g\left(\pm a\frac{\frac{x}{b^2}}{\sqrt{1-\frac{x^2}{b^2}}}\pm\mu\right)=0 \quad (40)$$

From first algebra equation for $\mu=0$ of previous system (40) is visible that $x=0$ is a root correspond to the equilibrium position, but there are also pair of the roots: $x_{1,3}=\pm b\sqrt{1-\left(ag\frac{1}{b^2\Omega^2}\right)^2}$ for $\frac{ag}{b^2\Omega^2}<1$. In this case for $\mu=0$ in system dynamics exists minimum a trigger of couped three singularities. Also, we can conclude that second algebra equation for $\mu\neq 0$ have minimum two roots. First approximation of the minimum vales of first two roots are $x_{1,3}\approx\pm 2\mu\frac{g}{\Omega^2}$, which correspond to the "one side right" and "one side left" equilibrium positions and with $x=0$ build a trigger of coupled two one side singularities appeared as a result of bifurcation by introducing Coulomb's type friction. By qualitative analyzing of the second algebra double equation from system (23) in the form: $\left(x\pm\mu\frac{g}{\Omega^2}\right)^2\left(R^2-x^2\right)=x^2\left(\frac{g}{\Omega^2}\mp\mu x\right)^2$, we conclude that appear also one trigger of coupled three triggers of coupled one side singularities.

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