OPTIMUM DESIGN OF MULTISPEED GEARBOXES AND MODELING OF TRANSMISSION COMPONENTS

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Abstract: *By applying the optimum design in the field of gear transmission design it is possible to define the optimal relations between the parameters of the complete gear transmission, and of each transmission stage separately. This paper presents a one criterion procedure for gear transmission optimization and multicriterion optimization procedure for each transmission stage. Second part of the paper is focused on modeling of cylindrical gears that are common used machine elements and main parts of gear transmissions. These models are made using part and assembly design module in CATIA V5R11 software. On the end of paper some applications of models in finite elements analysis and optimization are also described*

Keywords: *optimum design, multistage gearbox, computer added design, gears modeling, CATIA*

1. Introduction

Concept from optimization and decision theory can play an important role in all stages the design process. The optimizing design theory applying and methodology will be illustrated on a multispeed gearbox example. Gearboxes present a very important group of machine members, which are utilized in a great number of engineering fields and which must satisfy very rigorous technical requirements regarding reliability, efficiency, precise manufacturing of gears, bearing, etc. In addition, the latest achievements in the fields of technology and testing of the preciseness of manufacturing gears, bearings, etc., have been applied to the manufacturing process.

The development of the computer technology, together with the corresponding computer programs (AutoCAD, Solid Works, CATIA, etc.), have very quickly found their place in the development of the expert system for gearbox design at a high technical level. Thus, it can freely be said nowadays that the gearbox design is no longer a "routine job", which in most cases based upon the designer's experience and knowledge.

This paper demonstrates the application of a nonlinear multicriteria optimization method, with the purpose to build such a powerful method as a module into the gearbox design expert system. The introduction of some criteria considering the desirable performances, combined with high quality gearbox component modeling represents a significant step towards the reality of a gear train model.

2. Gearbox decomposition

Gearboxes represent complex mechanical systems that can be decomposed into the corresponding number of gears with corresponding interaction. This means that the procedure for multistage gearbox optimization can also be carried out through the corresponding number of stages. During the first optimization stage, characterized by comparatively small number variables, the distribution of transmission ratio per gearbox stages is defined from the conditions of the minimal volume of the gear sets. During the second stage, the multicriteria optimization problem is solved by introducing a greater number of criteria which represent the essential gearbox performances. Thereby, it is necessary to satisfy the restrictions from the following aspects: load distribution, stresses, kinematics and correct conjugate gear action.

The target function for multistage gearbox representing the volume of the gear sets can be written in the form the following relation [1]:

$$f(x) = 0.25\pi d_1^3 j_I((1+u_I^2)+j_{II}d_3^2/d_1^2 j_I(1+u_{II}^2)+...) \quad (1)$$

where:

u_I, u_{II} – the transmission ration for particular transmission stages of multistage gearbox;

d_1, d_3 – diameters of kinematics circles of the driver gears;

$j=b/d_1$ – ratio of width of the gear and diameter of the driver gear kinematics circle.

For the target function stated, it is also necessary to define the functional restrictions from the standpoint of the surface strength for the first stage of gearing, which can be written in the following form:

$$g'(x) = Z \cdot \frac{2 \cdot K \cdot T_1}{d_1^3} \cdot \frac{u_I + 1}{u_I} \leq \frac{[s_H]_I}{S_H} \quad (2)$$

and, from the standpoint of the volume strength:

$$g''(x) = K \cdot Y \cdot \frac{2 \cdot T_1}{\varphi_I \cdot d_1^2 \cdot m_I} \leq \frac{[\sigma_F]_I}{S_F} \quad (3)$$

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In the exactly analogous way, the functional restrictions from the standpoint of the surface and volume strength for other transmission stages of gearboxes are determined.

Commencing from the technical requirement concerning the transmission ratio of a gearbox, it is also necessary to determine the functional restriction in the form of the equation:

$$h_I(x) = u - u_I \cdot u_{II} \cdot \ldots \cdot u_N = 0 \tag{4}$$

Basing upon the determined target function and the restrictions, it can be noticed that this problem belongs to the field of nonlinear optimization with the restrictions in the form of inequalities. For the solution of this problem, the computer program SUMT, based on the mixed penalty functions, has been applied. Fig. 1 shows a graphic representation of the results of the computer program SUMT. Basing upon the section of the corresponding functions, the domains of the optimum transmission ratios for the multistage gearboxes are defined in the following way:

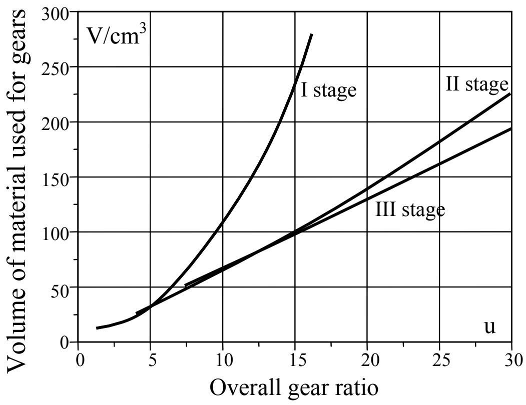

Figure.1: The relation between the volume of gear train and overall gear ratio.

To complete this analysis of decomposed gearbox, here are added a pair of restrictions in the form of inequalities, based on stress restrictions:

- tooth gear stress for I stage gear

$$g_3(\bar{x}) = \sigma_{F1} = K \cdot Y \frac{2 \cdot T_1}{\varphi_I \cdot d_1^2 \cdot m_I} \leq \frac{[\sigma_F]_I}{S_F} \tag{5}$$

- tooth-root gear stress for II stage gear

$$g_4(\bar{x}) = \sigma_{F3} = K \cdot Y \frac{2 \cdot T_1 \cdot u_I}{\varphi_{II} \cdot d_3^2 \cdot m_{II}} \leq \frac{[\sigma_F]_{II}}{S_F} \tag{6}$$

Based on gear stress relations the value of gear module is determinated:

- for contact stress

$$m \geq \left(\frac{2 \cdot K \cdot T_1 \cdot (u_I + 1) \cdot Z^2 \cdot S_H^2}{\varphi_I \cdot [\sigma_H]_I^2 \cdot u_I \cdot z_1^3} \right)^{\frac{1}{3}} \tag{7}$$

- for tooth-roth stress

$$m \geq \left(\frac{2 \cdot K \cdot T_1 \cdot Y \cdot S_F}{\varphi_I \cdot z_1^2 \cdot [\sigma_F]_I} \right)^{\frac{1}{3}} \tag{8}$$

Fig. 2 shows graphical interpretation of relations (7) and (8) in function of tooth number Z_1. Upper of two lines on the Fig. 2 presents values of gear module determinated on contact stress and lower one for values determinated on tooth-root stress. The lines and admissible space on Fig. 2. indicate that contact stress relation for gear module (7) is prior and is to be used for gear dimensions dermination.

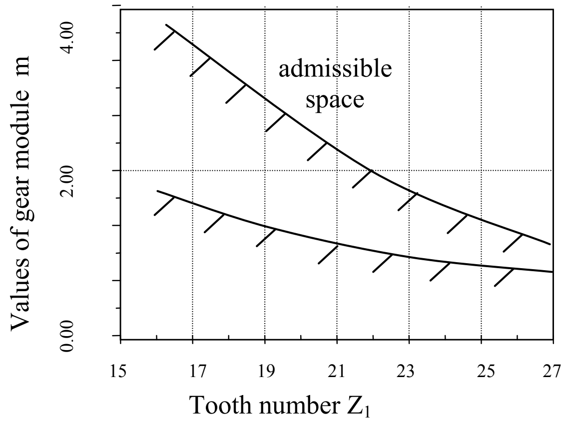

Figure. 2: Diagram of module values up to tooth number

3. Gears modeling

Gears are very important machine elements today and they are common used in different kinds of gearboxes and transmissions. Especially cylindrical gears are most applicable because of their very high efficiency and not complicated production. Modeling of cylindrical gears is very important process in machine design, as for making real model of gearbox, such for gear and transmission structure analysis and optimization. Last years this process can be done very fast and qualitative using new software tools such as **CATIA**. This software is very complex, but some main modules like **Part design and Assembly design** are in use for cylindrical gear modeling. The main problem in any gears modeling is to define a real gear tooth and after that to import it into gear body making. Cylindrical gears modeling consists of several phases, depends from gear body and kind of its production:

- The first phase of gear modeling is definition and making real involute gear teeth profile.
- The second phase, in case of cutting or pressed gear body, is to use Part design CATIA module to make gear body.
- The third phase, only in case of welding way made gear body, is to use Assembly design module to connect all its parts.

All this phases consists of several operations and it will be described separately in followed chapters.

Every chapter gives principal facts of general modeling, some special operations with advantages of using CATIA software in gears modeling and examples of different cylindrical gears that are modeled.

In analysis of internal and external gear profiles there are four different lines in one pitch, which defines complete profile of gear. So there are the involute profile arc, profile foot circle arc, addendum circle arc and trohoide arc as a connection [4]. In analytic-kinematics way for profile definition is to define a lot of restrictions and constrains for setting parameter equations each of this profile arcs and angles. After some matrix transformations matrix parameter equation for contact line of engaged gear tooth profiles can be determinated. Based on this analytic-kinematics model computer program is developed to define points of gear profiles [5].

Gears modelling is very useful and important, as to make real gear transmission simulation, so for lot of other analysis. Different software tools are in use today for machine design and machine elements modelling, as ACAD, Mechanical Desktop, Pro Engineer and last years Solid Works, CATIA etc. But it can be seen that gear modelling (especially internal gears) with real profiles is more complicated compared with modelling of all other machine elements. Here will be presented the possibilities of cylindrical gears modelling using CATIA V5R11 software. Depends of production way and form of gear body it is possible to use **Part design** module or **Assembly design** module of CATIA software.

For designing simplest cylindrical gear (flat) first step is to define correct sketch, where involute profile tooth coordinates (from first phase) should be imported. After that designer can apply Sketch based features (Create pad), to get cuted gear model as is shown at Fig. 3.

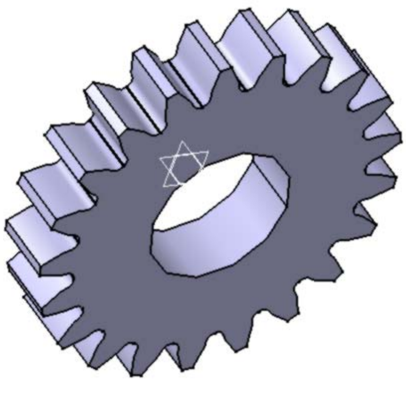

Figure 3: Simplest model of cylindrical gear

One step forward is designing a press made gear body, that could be modeled by rotating skatch made figure, or like simulation of production process. On Fig. 4 it is given a gear model made also by using skatch and few Sketch-Based, Dress-Up and Transformation Features. Presented gears are common in use and they have an external involute profile. But in some cases, like planetary gear train designing, it is necessary to make a model of internal profiled gear. For this purpose designer has to calculate a new table with involute profile coordinates, by using external gear as a tool for making internal profile. After that properly sketch and other features as for other cylindrical gears modelling has to be used.

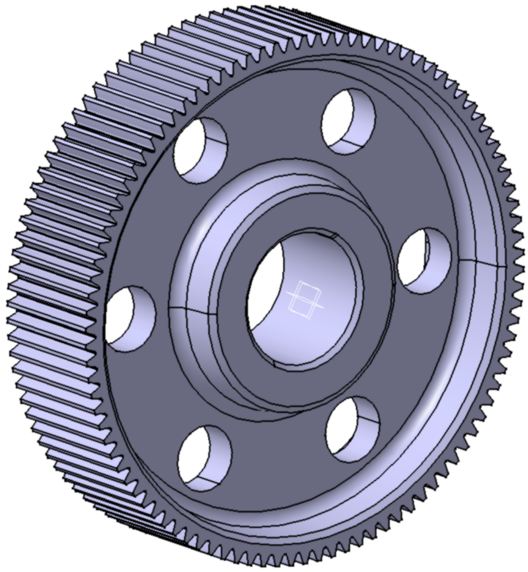

Figure 4: Press made model of cylindrical gear

Assembly design is another module in CATIA which is in use in aim to complete all parts and standard elements that are already modelled in Part or Shape design modules. Besides that it is possible to insert new bodies in existing assembly and also to do Boolean Operations between bodies if it is necessary. These Boolean operations between bodies are Assemble Bodies, Intersect Bodies, Add Bodies, Remove Bodies, Trim Bodies, Remove Lumps, etc.

The best sample of using Assembly design is cylindrical gear made by welding number of separated elements. It means that this type of gear consists of many elements that are modelled in Part design. The main part is outer plate with involute profiles that are welded with central cylinder with two circle plates and six stiffeners at both sides (Fig. 5).

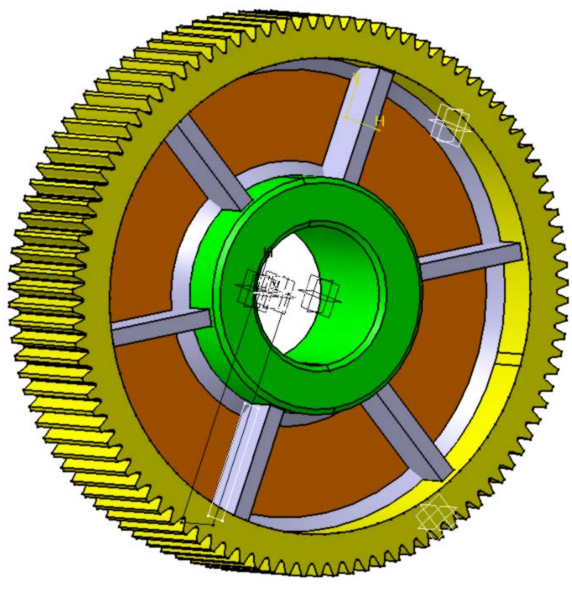

Figure 5: Cylindrical gear made by welding

A gear modeling is very significant because of many applications that could be done:

- After completing assembly it is possible to do kinematics simulations, using another CATIA module DMU.
- Internal and external gears models can be used for solving a lot of problems in mechanical engineering, such as **structural analysis**, contact pressure between corresponding gears and also thermal and many other analyses [8]. A typical example for this could be following structural analysis made using finite element method, where Fig. 6 shows gear model made of 77633 tetrahedrons which makes 18965 nodes.

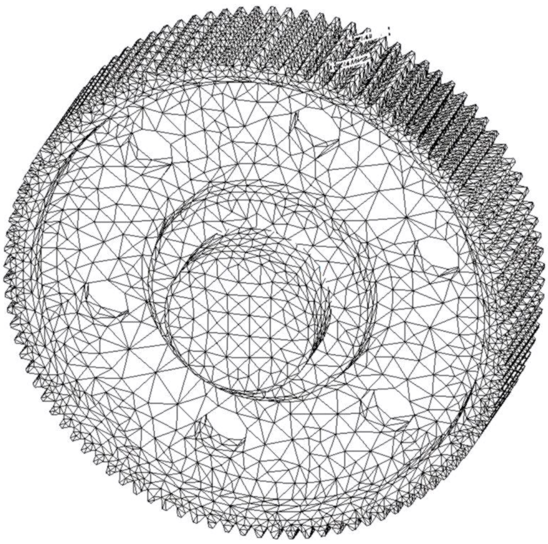

Figure 7: Gear model in form of finite element net

Stress values (Fig. 7) represent critical constructive points where gear is high loaded which could be also very useful in design and optimization process and procedure.

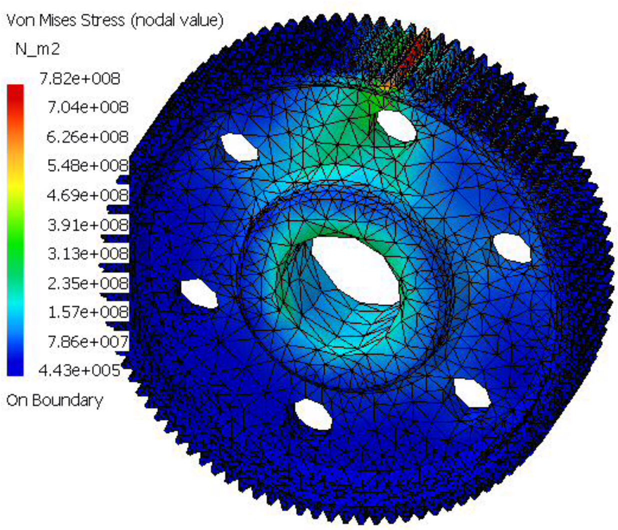

Figure 7: Stress values of loaded gear model calculated in structural analysis

4. Conclusion

The paper represents a brief illustration of a wider study undertaken with the aim of building the powerful multicriteria optimization methods into the expert system for gearbox design. It points out the necessity of decomposition multistage gearboxes as complex mechanical systems. In the way, the gearboxes optimization procedure is also carried out through the corresponding number of stages. In this first optimization stage, the domains of the practical application of gearboxes are defined, whereas, during the second stage, the multicriteria optimization problem is solved.

To resume the point of this modeling part of paper, here could be said that it presents only a brief of cylindrical gears modeling possibilities in CATIA software. Besides presentation of modeling in Part and Assembly design modules, at the end of this paper it is to add that CATIA is powerful and today may by completest design software in engineering with wide range of applications.

References

[1] Rosić B, 1993.: *Parameter Investigation and Optimization of Planetary Gear Train Transmission*, Ph.D Thesis, Mechanical Engineering Faculty, University of Belgrade

[2] Arora J.S, 1989.: *Introduction to optimum design*, McGraw–Hill Book Company, New York

[3] Rosić B., Marinković A.: *Planetary gear transmission as a tribosystem: Efficiency calculation and simulation*, ÖTG Jahres Symposium, Wien, November 2003.

[4] Colbourne, J. R., 1987: *The geometry of Involute gears*, Springer-Verlag, New York

[5] Rosić B., Rinkovec B., Marinković A., Pavlović N.: *The analytical-kinematics method for definition of internal cylindric gears,* Yugoslav Conference “IRMES 2002”, Faculty of Mechanical Engineering Srpsko Sarajevo, Jahorina – BIH, September 2002, Proceedings, pp. 625-630.

[6] Rosić B.: *Planetary gear trains*, Monography, Faculty of Mechanical Engineering, University of Belgrade, edited in year 2003.

[7] Rosić B., Marinković A., Vencl A, 2004.: *Cylindrical Gears modeling using CATIA software,* 4th International Conference “RADMI 04“, Zlatibor, Serbia and Montenegro, August-September 2004., Proceedings on CD, pp. 73-77.

[8] Rosić B., Marinković A., Vencl A, 2004.: *Modeling and Structural Optimization of Cylindrical Gears construction profiles,* Yugoslav Conference “IRMES 04”, Faculty of Mechanical Engineering Kragujevac, Kragujevac, September 2004, Proceedings, pp. 173-178.